# MURANG’A UNIVERSITY OF TECHNOLOGY

## SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**FOURTH** YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA2306– LINEAR ALGEBRA II

DURATION: 2 HOURS

DATE: 14TH DECEMBER 2017

TIME: 2.00PM – 4.00PM

**Instructions to Candidates:**

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

**SECTION A (Compulsory)**

**QUESTION ONE (30 Marks)**

a) i) Given that $u$ and $v$ belong to a vector space $V$, simplify the expression

$E = 3(2u - 4v) + 5u + 7v$. (1 Mark)

ii) Express the vector $(1, -2,5)$ in $R^3$ as a linear combination of the vectors $u_1 = (1,1,1)$, $u_2 = (1,2,3) and\ u_3 = (2, -1,1)$. (4 Marks)

b) i) The mapping $F: R^3 \rightarrow R^2$ is defined by $F(x, y, z) = (yz, x^2)$. Find $F(2,3,4)$. (1 Mark)

ii) Show that the mapping $G: R^2 \rightarrow R^2$ defined by $G(x, y) = x + y, x)$ is linear. (4 Marks)

c) Given that $H: R^2 \rightarrow R^2$ is the linear operator defined by $H(x, y) = (2x + 3y, 4x - 5y)$, find the matrix representation of $H$ relative to the basis $S = \{u_1, u_2\} = \{(1,2), (2,5)\}$. (5 Marks)

d) Using cofactor expansion compute the determinant of

$$A = \begin{pmatrix} 5 & -2 & 2 & 7 \\ 1 & 0 & 0 & 3 \\ -3 & 1 & 5 & 0 \\ 3 & -1 & -9 & 4 \end{pmatrix}$$ (4 marks)

e) Given that $B = \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix}$ find $F(B)$ where:

i. $F(t) = t^2 - 3t + 7$ (3 Marks)

ii. $F(t) = t^2 - 6t + 13$ (3 Marks)

f) Find the characteristic and minimal polynomial of;

$$C = \begin{pmatrix} 3 & 2 & -1 \\ 3 & 8 & -3 \\ 3 & 6 & -1 \end{pmatrix}$$ (5 marks)

**SECTION B (Answer any two questions)**

**QUESTION TWO (20 Marks)**

a) Express the polynomial $P(t) = t^2 + 4t - 3$ in the vector space V of real polynomials as a linear combination of the polynomials $P_1 = t^2 - 2t + 5, P_2 = 2t^2 - 3t\ and$ $P_3 = t + 3$. (6 Marks)

b) $V'$ is the vector space of n-square real matrices and M is an arbitrary but fixed matrix in $V'$. Given that $F: V' \rightarrow V'$ is defined by $F(A) = AM + MA$ where $A$ is a matrix in $V'$, show that F is linear. (4 Marks)

c) Given the linear mapping $G: R^2 \to R^2$ defined by $G(x, y) = (3x + 4y, 2x - 5y)$ and the bases $E = \{e_1, e_2\} = \{(1,0), (0,1)\}$ and $S = \{u_1, u_2\} = \{(1,2), (2,3)\}\ of\ R^2$:

i. Find the matrix A representing G relative to the basis E. (3 Marks)

ii. Find the matrix B representing G relative to the basis S. (7 Marks)

**QUESTION THREE (20 Marks)**

a) Find all $t$ such that:

i. $\begin{vmatrix} t-4 & 3 \\ 2 & t-9 \end{vmatrix} = 0$ (2 Marks)

ii. $\begin{vmatrix} t-5 & 4 & 0 \\ -1 & t & 0 \\ 0 & 0 & t-4 \end{vmatrix} = 0$ (2 Marks)

b) Show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y - x)(z - x)(z - y)$ (4 Marks)

c) Given that $B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{pmatrix}$, find;

i. $|B|$ (2 Marks)

ii. $adj\ B$ (5 Marks)

iii. $B^{-1}$using $adj\ B$ (2 Marks)

d) Using the result of (iii) above, solve the system of equations

$$x + y + z = 2$$
$$2x + 3y + 4z = 1$$
$$5x + 8y + 9z = 3$$

(3 Marks)

**QUESTION FOUR (20 Marks)**

a) Define the terms eigenvalue and eigenvector. (2 Marks)

b) Given that $A = \begin{pmatrix} 6 & 16 \\ -1 & -4 \end{pmatrix}$, verify that $X = \begin{pmatrix} -8 \\ 1 \end{pmatrix}$ is an eigenvector of A with a corresponding eigenvalue of $\lambda = 4$ (4 Marks)

c) Given that $B = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

i. Find all eigenvalues and corresponding eigenvectors of matrix B (8 Marks)

ii. Find a nonsingular matrix P and its inverse ($P^{-1}$)such that $D = P^{-1} A\ P$ is diagonal. (3 Marks)

iii. Find $B^6$. (3 Marks)

**QUESTION FIVE (20 Marks)**

a) i) State Cayley-Hamilton theorem for a square matrix A. (1 Mark)

ii) Determine the characteristic polynomial of the $2 \times 2$ matrix $A = \begin{pmatrix} a & b \\ c & b \end{pmatrix}$ and verify Cayley-Hamilton theorem for matrix A. (7 Marks)

b) Determine the eigenvectors corresponding to each eigenvalue and a basis for the eigenspace corresponding to each eigenvalue for the matrix

$B = \begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$ (12 marks)