



# MURANG'A UNIVERSITY OF TECHNOLOGY

## SCHOOL OF PURE AND APPLIED SCIENCES

### DEPARTMENT OF APPLIED SCIENCES

#### UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN MATHEMATICS AND ECONOMICS**

AMS 321 – THEORY OF SAMPLING TECHNIQUES

DURATION: 2 HOURS

DATE: 7<sup>TH</sup> DECEMBER, 2017

TIME: 2.00 – 4.00 P.M.

#### **Instructions to Candidates:**

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

## SECTION ONE - COMPULSORY

### QUESTION ONE

- (a) Consider a population of five units ( $N=5$ ) taking values 0,2,11,5,21. Assume that all possible simple random samples of size 2 are taken hence compute
- i) Sample mean ( $\bar{x}$ ) (2 marks)
  - ii) Expected value of  $\bar{x}$  (2 marks)
  - iii) Var ( $\bar{x}$ ) (3 marks)
- (b) Define the following terms as used in theory of sampling technique
- i) Sampling unit
  - ii) Sampling frame
  - iii) Sample survey (3 marks)
- (c) Signatures to a petition were collected on 676 sheets. Each had enough space for 42 signatures but on many sheets, a small number of signatures had been collected. The number of signatures per sheet were counted on a random sample of 50 sheets with results tabulated below
- |       |    |    |    |    |    |    |    |    |    |    |   |   |   |   |   |   |    |    |
|-------|----|----|----|----|----|----|----|----|----|----|---|---|---|---|---|---|----|----|
| $y_i$ | 42 | 41 | 36 | 32 | 29 | 23 | 19 | 15 | 11 | 10 | 9 | 7 | 6 | 5 | 4 | 3 | 16 | 14 |
| $f_i$ | 23 | 4  | 1  | 1  | 1  | 1  | 1  | 2  | 1  | 1  | 1 | 1 | 3 | 4 | 1 | 1 | 2  | 1  |
- i) Estimate the total number of signatures (3 marks)
  - ii) The mean of the total number of signatures (3 marks)
- (d) State and explain at least two properties of estimates (4 marks)
- (e) Given that you have a population of  $N$  units and a sample of  $n$  units chosen from this population. Show that  $\bar{y}$  is an unbiased estimator. (5 marks)
- (f) If  $N=6$  and  $n=3$  and the values of the population are 1,2,4,6,7 and 16 find the standard error of the mean (5 marks)

## SECTION TWO – ANSWER ANY TWO

### QUESTION TWO

- (a) Suppose that the following summarized information is made available  $n=25$ ,  $N=275$ ,  $\bar{x}=9.2$ ,  $\bar{y} = 2.6$ ,  $\sum_{i=1}^{25} x_i y_i = 500$ ,  $\sum_{i=1}^{25} y_i^2 = 170$  Estimate  $R$  and  $\text{var}(R)$  (6 marks)
- (b) Sample survey experience the problem of estimating population characteristics from a subset (sample). List five reasons as to why we use a sample (5 marks)

- (c) A survey of food costs is performed by taking a simple random sample of 48 basic food stuffs purchased in a supermarket. Prices in dollars are recorded in two separate occasions, three months apart denoted by  $x_i$  and the latter  $y_i$  respectively.  $r = \frac{\bar{y}}{\bar{x}}$  gives an indication of change of these basic food prices over three months period in the form of an estimate of the population ratio  $R$  of the mean prices of food stuffs on the two occasions. The following results were obtained:  $n = 48, \sum y_i^2 = 9270.6, \bar{y} = 12.07, x_i^2 = 8431.7, \sum x_i y_i = 8564.1, \bar{x} = 11.41$  Estimate the sampling variance of the ratio estimator and the 95% confidence interval for  $R$ . (9 marks)

### QUESTION THREE

- (a) In a private library, books are kept on 130 shelves of similar size. The numbers of books on 15 shelves picked at random were found to be 28, 23, 25, 33, 31, 18, 22, 29, 30, 22, 26, 20, 21, 28 and 25.
- Estimate the total number  $Y_T$  of books in the library (4 marks)
  - Calculate an approximate 95% confidence interval for  $Y_T$  (12 marks)
- (b) Survey may be described as descriptive or analytical depending on the objectives. Explain (4 marks)

### QUESTION FOUR

- (a)  $\bar{y}_w$  is an unbiased estimator of  $\bar{Y}$  in the context of stratified random sampling. Proof (4 marks)
- (b) The following data shows the stratification of some farms in some country

Stratum	1	2	3	4	5
$N_i$	45	41	74	98	17
$\bar{y}_i$	9.6	9.1	11.2	6.9	7.3
$S_i^2$	1.74	1.96	1.13	2.03	1.31

- Calculate the overall mean and variance (10 marks)
- For a stratified simple random sample of size 80, determine the appropriate stratum sample size under proportional allocation and Neyman allocation (6 marks)

### QUESTION FIVE

- (a) A fashion designer has shops in 5 regions. The regions have 12 shops and the other two have 8. To estimate total sales  $Y_T$  of clothes sold from the shops in a particular week, the manager calls

12 shops chosen by picking 3 regions at random then making a choice of 3,5 and 4 shops from the chosen regions. The results were as follows

Region 1 260 296 182 - 12 shops  
 Region 2 156 261 302 230 8 shops  
 Region 3 196 356 284 268 12 shops

(8 marks)

- (b) Public works sector carried out a survey to investigate the extent of absenteeism not connected with illness or official holidays. A random sample of 1000 men out of a total of 36,000 are asked how many days they have taken off work, in the previous 6 months as casual holidays. The results were as follows:

Day off	0	1	2	3	4	5	6	7	8	9
No. of men	451	162	187	112	49	21	5	11	2	0

- i) Obtain an estimate of  $\bar{X}$  the average number of days 'casual holiday' taken by the work men in the industry (4 marks)
- ii) Obtain the estimate of  $S^2$  (4 marks)
- iii) Using the normal approximation to the distribution of sample mean obtain an approximate 95% systematic two sided confidence interval for  $\bar{X}$  (4 marks)