MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF APPLIED SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

THIRD YEAR **FIRST** SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS & PROGRAMMING

AMS 309 – STOCHASTIC PROCESS I

DURATION: 2 HOURS

DATE: 7TH DECEMBER, 2017

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION ONE - COMPULSORY

QUESTION ONE

(a) Define the following terms

i) Stochastic process

ii) Ergodicity (2 marks)

(b) Discuss the memorylessness property and fresh start property of the bernoulli process

(2 marks)

(c) State and explain the postulates of the poisson process (3 marks)

(d) Consider the following markov chain with state space s = {1,2,3,4}

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Classify state 2 as persistent or transient (5 marks)

(e) Give three limitations of queuing theory (3 marks)

(f) You get emails according to a poisson process at the rate of $\lambda = 0.2$ messages per hour. You check your email every hour

i) What is the probability of finding 0 messages? (2 marks)

ii) What is the probability of finding 1 message? (2 marks)

iii) Suppose you have not checked your email for a whole day. What is the probability of finding no new messages? (2 marks)

(g) Let Y_k be the k^{th} arrival time be equal to the sum $Y_k=T_1+T_2+\ldots+T_k$ of k independent …. Geometric random variables. Show that the Probability Mass Function of Y_k is given by

$P_{yk}(t) = \binom{t-1}{k-1} P^k (1-p)^{t-k}$ t = k, k+1 …. (5 marks)

(h) Let $\{Xn\ n \geq 0\}$ be a Markov chain with three sates S={0,1,2}. The transition probability matrix P is given as

$$P = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & \frac{7}{10} & \frac{1}{5} \end{bmatrix}$$

Determine the following probabilities

i) $P(X_1 = 1 \mid X_0 = 2)$ (1 mark)

ii) $P(X_2 = 2 \mid X_1 = 1)$ (1 mark)

iii) $P(X_2 = 2 \mid X_0 = 2) \,.\, P(X_1 = 1 \mid X_0 = 2)$ (2 marks)

SECTION TWO – ANSWER ANY TWO

QUESTION TWO

(a) The computer lab at a public university has a help desk to assist students working on computer spreadsheet assignments. The students patiently form a single line in front of desk to wait for help. Students are served based on first come first served basis. On average fifteen students per hour arrive at the help desk according to poisson distribution. The help desk serve can help an average of twenty students per hour according to exponential distribution. Calculate:

i) The average number of students in the system (2 marks)

ii) The average number of students waiting in line (2 marks)

iii) The average time a student spends in the system (2 marks)

iv) The average time a student spends waiting in line (2 marks)

v) The probability of having four students in the system (2 marks)

(b) The polya process is a non-stationary pure birth process with birth rate λn depending on time i.e.

$\lambda n = \frac{1+an}{1+at}$

Show that the difference differential equations are given as

$$P_n^1(t) = -\left(\frac{1+an}{1+at}\right) P_n(t) + \left(\frac{1+a(n-1)}{1+at}\right) P_{n-1}(t)\ n \geq 0$$

$$P_0^1(t) = -\left(\frac{1}{1+at}\right) P_0(t)$$ (10 marks)

QUESTION THREE

Consider the following probability transition matrix of the state space s =(1,2,3)

$$P = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Classify the states (20 marks)

QUESTION FOUR

(a) Find the limiting probabilities of the following stationaary distribution of a Markov chain

$$P = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$ (5 marks)

(a) The golden muffler shop has decided to open a second garage bay and hire a second mechanic to handle installations. Customers arrive at the rate of two per hour according to poisson distribution. Each mechanic installs mufflers at the rate of three per hour according to exponential distribution. Find

i) The probability that there are no customers in the system

ii) The average number of customers waiting in line

iii) The average number of customers in the system

iv) The average time a customer spends waiting in line

v) The average time a customer spends in the system (10 marks)

(b) The number of customers N(t) arriving at a bank in an interval duration t follows poisson distribution (mean λt). If the rate of arrival is three per minute, find the probability of the number of customers in every two minutes in

i) Exactly four arrivals (2 marks)

ii) Greater than four arrivals (3 marks)

QUESTION FIVE

(a) In the pure birth process, the probability that k events occur between t and tth given that n events occurred to epoch t is given as shown below. Find the difference differential equations for the process when $\lambda_n = n\lambda$

$$P_k(h) = \begin{cases} \lambda h + 0(h) & K = 1 \\ 0(h) & K \geq 2 \\ 1 - \lambda h + 0(h) & K = 0 \end{cases}$$ (10 marks)

(b) The probability of a dry day (state 1) following a rainy day is $\frac{1}{3}$ and a rainy day following a dry day is $\frac{1}{2}$

i) Find the transition probability matrix of the Markov chain (2 marks)

ii) Determine the probability of May 3rd is a dry day if May 1st is a dry day (3 marks)

iii) Determine the probability of May 5^{th} is a dry day in May 1^{st} is a dry day. (3 marks)

(c) Dave fails quizzes with probability $\frac{1}{4}$ independently of other quizzes. What is the probability that Dave fails exactly two of the next six quizzes? (2 marks)