



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF APPLIED SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING**

AMS 202 – PROBABILITY & STATISTICS II

DURATION: 2 HOURS

DATE: 15TH DECEMBER, 2017

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION ONE - COMPULSORY

QUESTION ONE

- (a) State any two properties that a probability density function of continuous random variable x should satisfy (2 marks)
- (b) Suppose that a random variable x has probability density function given by

$$f(x) = \begin{cases} 1/9 & (3 + 2x + x^2), 0 \leq x \leq 3 \\ 0 & \text{Otherwise} \end{cases}$$

Determine the mean and variance of x (6 marks)

- (c) A random variable x is defined to have a poisson distribution with p.d.f.

$$f(x) = \begin{cases} \frac{e^{-\theta} \theta^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & \text{Otherwise} \end{cases}$$

Determine the mean of this distribution (4 marks)

- (d) The incident of occupational disease in an industry if that the workers have a 20% chance of suffering from it. What is the probability that out of the six workers five or more contract the disease (4 marks)
- (e) A child welfare officer asserts that the mean sleep of young babies is 14 hours a day. A random sample of 100 babies show that their mean sleep was only 13 hours 40 mins with a standard deviation of 3 hours. At 5% level of significant test the assertion that the mean sleep of babies is less than 14 hours a day (4 marks)
- (f) In 500 independent calculations, a student made 25 errors. His instructor randomly checked seven calculations of the student. Find the probability that an instructor detects exactly 2 errors (4 marks)
- (g) A continuous random variable x has moment generating function $\mu(t) = e^{2t^2+3t}$. Determine the
- i) $E(x)$ (2 marks)
- ii) $\text{Var}(x)$ (4 marks)

SECTION TWO – ANSWER ANY TWO QUESTIONS

QUESTION TWO

- (a) For a random variable x having probability generating function $G(t)$, show that the expected value $E(x)$ and the variance (x) are given by $G'(1)$ and $G''(1) + G'(1) - [G'(1)]^2$ respectively (7 marks)
- (b) The distribution of the annual salaries in US dollars of data collection working in Kenya is a continuous random variable x given by probability density function

$$f(x) = \begin{cases} kx(9-x) & 0 \leq x \leq 3 \\ 0 & \text{Otherwise} \end{cases}$$

Where k is a constant

Determine

- i) The value of k (3 marks)
- ii) The mean salary of the data collection (4 marks)
- iii) The standard deviation of the distribution of salary of the data collection (6 marks)

QUESTION THREE

- (a) A continuous random variable x has a pdf $f(x)$ given

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x) & 0 \leq x \leq 4 \\ 0 & \text{Otherwise} \end{cases}$$

Determine the mode of the random variable x (5 marks)

- i) Given that x is a random variable from a binomial distribution with parameter (n,p) , show that the expected value and variance of x is np and npq respectively (11 marks)
- (b) Company XYZ prepares this for jam, which are labeled to have a mass of 500 grams. During the preparation the tins are set to have a mean mass of 550 grams with a standard deviation of 20 grams. One tin is selected from a large batch and is found to have a mass of 500 grams. Assuming the masses are normally distributed, test whether the mass of this tin is significantly different from 550 gram at 5% level of significance (4 marks)

QUESTION FOUR

- (a) Let x be a continuous random variable having pdf $f(x)$ given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } 0 \leq x < \infty \\ 0 & \text{Otherwise} \end{cases}$$

Determine the mean and variance of x (8 marks)

- (b) If x is a discrete random variable with a and b being constants, show that $\text{Var}(ax+b) = a^2\text{Var}(x)$ (4 marks)

If x is a continuous random variable x with pdf given by

$$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{1(\alpha)\beta^\alpha} & \text{for } x \geq 0, \beta > 0, \alpha > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Obtain the moment generating function of x , hence determine the mean of x (8 marks)

QUESTION FIVE

- (a) A computer battery have a life time of T hours with a pdf $f(t) = \begin{cases} \frac{1}{400} e^{-\frac{t}{400}}, & t > 0 \\ 0 & \text{Otherwise} \end{cases}$

Determine the probability that the battery

- i) Fails before 30 hours (3 marks)
 - ii) Life is between 35 and 50 hours (3 marks)
 - iii) Life is greater than mean life (3 marks)
- (b) A die is rolled until a six appears. Let x be the number of trials until a six appears. Find the mean of x using probability generating function (6 marks)
- (c) A continuous random variable x is uniformly distributed over the interval $[a,b]$. use the moment generating function to determine mean of x (5 marks)