MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF APPLIED SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FIRST YEAR **FIRST** SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH PROGRAMMING)

AMM 2101 – FOUNDATION MATHEMATICS I

DURATION: 2 HOURS

DATE: 6TH DECEMBER, 2017

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION ONE - COMPULSORY

QUESTION ONE (30 MARKS)

(a) In how many ways can ten persons arrange to sit at a round table? (2 marks)

(b) A committee of nine persons is to be selected from thirteen men and eight women. In how many ways can this be done if the committee is to have at least six women? (4 marks)

(c) Show that $(p \vee q) \wedge [(\neg p) \wedge (\neg q)]$ is a contradiction (3 marks)

(d) Show that 0.535353… is a rational number (3 marks)

(e) Show that the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2 + 3$ is not injective (5 marks)

(f) Use the principle of mathematical induction to prove that

$1^2 + 2^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ (5 marks)

(g) Construct the complete truth table for $(p \wedge q) \leftrightarrow r$ (4 marks)

(h) Find the principle value of $ln(3+7i)$ (4 marks)

SECTION TWO – ANSWER ANY TWO QUESTIONS

QUESTION TWO

(a) Given that $\cos\theta = \frac{3}{5}$ *and* $\sin\alpha = \frac{24}{25}$ where α and θ are acute angles, find the value of $\sin(\theta + \alpha)$ without finding the values of α and θ (5 marks)

(b) Solve the equation $Cos(2x) - 9\,Sin(x) + 5 = 0$ *for* $0 \leq x \leq 360°$ (6 marks)

(c) Express $3Cos\,\theta - 7Sin\,\theta$ into the form $R\,Cos(\theta + \alpha)$ where R and α are constants hence solve the equation $3Cos\,\theta - 7Sin\,\theta = 2$ for 0 *for* $0 \leq \theta \leq 360°$ (9 marks)

QUESTION THREE

(a) Prove that $(n+2)! + (n+1)! + n! = (n+2)^2 n!$ (4 marks)

(b) Find the values of n that satisfy the equation $14({}^{n}P_3) = {}^{(n+2)}P_4$ (7 marks)

(c) Prove that ${}^{n}C_r = {}^{n}C_{(n-r)}$ where $r, n \in \mathbb{N}$ and $r \leq n$ (3 marks)

(d) How many even numbers greater than 2000 can be formed with the digits 1,2,4 and 8 if the digits are not to be repeated in a given number (6 marks)

QUESTION FOUR

(a) State the two De-Morgan's Laws hence prove that $(p \vee q) \vee (\neg p) \wedge (\neg q)$ is a tautology (4 marks)

(b) Use the principle of mathematical induction to prove that $23^n - 1$ is divisible by 11 $\forall n \epsilon \mathbb{N}$ (6 marks)

(c) Define a function $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = 5x + 9$ show that $f(x)$ is bijective hence find its inverse (7 marks)

(d) State the inverse and the converse of the statement "if there is fire then oxygen is present" (3 marks)

QUESTION FIVE

(a) Given that $S(0) = 1$, show that $S(b) = b + 1$ $\forall n \epsilon \mathbb{N}$ where $S(x)$ denote the successor of x $\forall n \epsilon \mathbb{N}$ (3 marks)

(b) Let $a, b \epsilon \mathbb{Z}$ and $n \in \mathbb{N}$ prove that if $a \equiv b(mod n)$ then $a^2 \equiv b^2(mod n)$ (5 marks)

(c) A survey in a town showed that 12,000 people were smokers and 5,000 drunk alcohol. There were 3,000 people who smoke and drunk alcohol. Determine:

i) The number of persons who drink alcohol but do not smoke

ii) The number of persons who smoke but do not drink alcohol

iii) Number of persons who either smoke or drink alcohol (3 marks)

(d) Find all the cub-roots of the complex number $Z = 5 + 4i$ (9 marks)