MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF APPLIED SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

THIRD YEAR **FIRST** SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/ BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMM 308 – SANITATION AND WASTE CONTROL AND MANAGEMENT

DURATION: 2 HOURS

DATE: 11TH DECEMBER, 2017

TIME: 9.00 – 11.00 A.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION ONE - COMPULSORY

QUESTION ONE

(a) Given that $Z_i = 1 - 3_i \; and \; Z_2 = -2 + 5i$, determine $\frac{Z_1 Z_2}{Z_1 + Z_2}$ (4 marks)

(b) Evaluate the limits

i) $\lim_{Z \to 2} \frac{Z^2+3}{iZ}$ (2 marks)

ii) $\lim_{Z \to 2i} \frac{Z^2+4}{2Z^2+Z-6i}$ (3 marks)

(c) Express the function $\omega = \frac{Z^2+1}{Z}$ in the form $\omega = u(x,y) + iv(x,y)$ (5 marks)

(d) Find the derivative of $f(Z) = \frac{Z^3 - 2Z}{Z+1}$ (3 marks)

(e) Show that $f(Z) = 3\bar{Z}$ is not analytic (2 marks)

(f) Evaluate $\int Z^2 dZ$ along the curve c given by $x = t^2$ and $y = t$ between $t = 1$ and $t = 2$

(4 marks)

(g) Evaluate $\int \frac{e^z dz}{Z^2-9}$ around the circle of radius 2 units centred at Z=0 (3 marks)

(h) Evaluate $\oint \frac{Cos\pi Z^2}{(Z-1)(Z-2)}$ around the circle given by $|Z| = 3$ (4 marks)

SECTION TWO – ANSWER ANY TWO QUESTIONS

QUESTION TWO

(a) Given that $Z = 1 - i$, and $f(Z) = Z^2 + 2Z + 1$, evaluate $|f(Z)|$ and $Arg[f(Z)]$ (6 marks)

(b) Use DeMoivre's theorem to expand Cos30 in terms of Cosθ (5 marks)

(c) Find all the cube roots of the complex number $Z = 1 + i$ and represent them on the Argand diagram (9 marks)

QUESTION THREE

(a) Find a solution to the equation $Sin\, Z = i$ (5 marks)

(b) Evaluate $\lim_{Z \to} e_3^{\pi} i \; \frac{Z(Z-e^{\pi y\, 3})}{Z^2+1}$ giving your answer in the form $x + iy$ (5 marks)

(c) Find $u(x,y)$ and $r(x,y)$ such that $ZSinhZ = u + iv$ where $Z = x + iy$ and x, y, u and v are real (5 marks)

(d) Locate and name all the singularities of the function $\frac{(Z+3i)^5}{(Z^2-2Z+5)^2}$ (5 marks)

QUESTION FOUR

(a) Prove that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and find a function v such that $f(Z) = u + iv$ is analytic. Express *f(Z)* in terms of Z. (8 marks)

(b) State and prove Cauchy's integral theorem. (5 marks)

(c) Evaluate $\int \frac{1}{Z-Z_o} dz$ along a simple closed contour C having the origin as an interior point

(7 marks)

QUESTION FIVE

(a) Using Laurent's Theorem, prove that $\oint f(Z)dZ = 2\pi iR,$ where *f(Z)* is analytic inside and on a simple closed curve C except at one point Z=a inside C, R being the residue at Z=a

(6 marks)

(b) Find the residues of

i) $f(Z) = \left(\frac{Z+1}{Z-1}\right)^2$ at Z=*1* (3 marks)

ii) $f(Z) = Z^{-2}CotZ$ at the origin (4 marks)

(c) Using the Cauchy integral formulae, find the values of:

i) $\oint \frac{Sin\,6_Z}{Z-{}^{\pi}/_{6}} dZ$ (3 marks)

ii) $\oint \frac{Sin\,6_Z}{(Z-{}^{\pi}/_{6})^3} dZ$ (4 marks)