MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

THIRD YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMM301 – REAL ANALYSIS

DURATION: 2 HOURS

DATE: 11TH DECEMBER 2017

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A (Compulsory)

QUESTION ONE (30 Marks)

a) Define the following terms

i. Bounded sets

ii. Closed sets

iii. Convergent sequence (6 Marks)

b) Given the set $S = \left\{\frac{m}{n} : \text{m! n} \,\epsilon \mathbb{N}, m < n\right\}$. Find the infimum and supremum of S. (2 Marks)

c) Show that the set of natural numbers $\mathbb{N}$ is not open. (3 Marks)

d) Show that $\lim_{n\to\infty} \frac{1}{n} = 0$ by first principles method. (4 Marks)

e) Prove that every convergent sequence of real numbers is bounded. (5 Marks)

f) Find the sum of the following series

$\sum_{n=1}^{8} \frac{1}{3^{n+1}}$ (4 Marks)

g) Let $f(x) = x^2 + 1$ for all $X \in \mathbb{R}$. Prove that

$\lim_{x\to 2} f(x) = 5$ (6 Marks)

SECTION B (Answer any two questions)

QUESTION TWO (20 Marks)

a) Show that $\sqrt{7}$ is not a rational number (5 Marks)

b) If X and Y are sets. Show that;

i. $(X \cup Y)^c = X^c \cap Y^c$ (6 Marks)

ii. $(X \cap Y)^c = X^c \cup Y^c$ (6 Marks)

c) Prove that for all $x \in \mathbb{R}$ such that $x > 0$ there exist $x^{-1} > 0$ (3 Marks)

QUESTION THREE (20 Marks)

a) Show that the sequence (x_n) in $\mathbb{R}$ has unique limit. (5 Marks)

b) Let X be a non empty set and define;

$$d(x,y) = \begin{cases} 0, if & x = y \\ 1, if & x \neq y \end{cases}$$

Show that d is a metric on X. (5 Marks)

c) Show that a real number P is a limit point of a subset S of $\mathbb{R}$ if and only if every neighborhood of P contain infinitely many points of S. (7 Marks)

d) Show that the union of any finite number of closed sets is closed. (3 Marks)

QUESTION FOUR (20 Marks)

a) Prove that the set $\mathbb{R}$ of real number is uncountable. (6 Marks)

b) Determine whether the following series conveys or diverges

$\sum \frac{2^n+5}{3^n}$ (4 Marks)

c) Prove that $f(x) = x^2 + 2x + 6$ is continous at $x = 3$ (6 Marks)

d) Evaluate:

i. $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$ (2 Marks)

ii. $\lim_{x\to 2} \frac{x^2-4}{x-2}$ (2 Marks)

QUESTION FIVE (20 Marks)

a) Show that $\lim_{n\to\infty} \left(\frac{3n+2}{n+1}\right) = 3$ (3 Marks)

b) Evaluate the following;

$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$ (4 Marks)

c) Show that if x and y are real numbers with $x < y$ then there exists an irrational number w such that $x < w < y$ (4 Marks)

d) Calculate the limit superior and limit inferior for the following sequences

i. $x_n = (-1)^n$ (3 Marks)

ii. $x_n = n(1 + (-1)^n)$ (3 Marks)

e) Let M and N be neighborhood of a point x. Show that $M \cap N$ is also a neighborhood of x.

(3 Marks)