MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

SECOND YEAR **FIRST** SEMESTER EXAMINATION FOR BSC APPLIED STATISTICS WITH PROGRAMMING AND BSC. MATHEMATICS AND ECONOMICS

AMM104 – CALCULUS II

DURATION: 2 HOURS

DATE: 7TH DECEMBER 2017

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A (Compulsory)

QUESTION ONE (30 Marks)

a) Given the implicit function $x^2y^3 + y^2 - x^4 = 5$, find $\frac{dy}{dx}$ (4 Marks)

b) A curve is defined by the parametric equations

$$y = t^2 + 3t - 4$$
$$x = 5t + 2$$

Find its Cartesian equation hence find $\frac{dy}{dx}$. (3 Marks)

c) Find $\frac{dy}{dx}$ given that $y = \frac{\cosh\ x}{\tanh\ (2x)}$ (3 Marks)

d) Evaluate

$\int_0^{\pi/6} \cos\theta\ sin^2\theta d\theta$ (3 marks)

e) Define $z = f(x.y) by\ z = 4x^2y + x^3 - y^2$. Find $\frac{dz}{dx}$ along the curve $y = x^2 + 3$

(5 Marks)

f) Find

$\int \frac{2x+3}{(x^2+3x-2)^3} dx$ (2 marks)

g) Evaluate the following double integral

$I = \int_0^1 \int_1^3 (2x^2y - 4y^2) dydx$ (3 marks)

h) Find $\frac{dy}{dx}$ given that $y = \cosh^{-1}(x/2)$ (4 marks)

i) Find $\int (x^2 + 2)\ Cos(2x)dx$ (3 marks)

SECTION B (Answer any two questions)

QUESTION TWO (20 Marks)

a) Solve the equation;

$7\sinh(x) + 20\cosh x = 24$ (5 marks)

b) Find $\frac{dy}{dx}$ given that $y = \tanh^{-1}\left(\frac{1-x}{1+x^2}\right)$. (6 Marks)

c) A curve is defined by parametric equations;

$$y = t^2 + 2t - 3$$
$$x = e^{3t} + 4$$

Find $\frac{dy}{dx}$. (3 Marks)

d) Find the equations of the normal and tangent to the curve;

$x^2y + xy - y^2 + 3 = 0$

At a point $(1,3)$. (6 Marks)

QUESTION THREE (20 Marks)

a) Define $z = f(x, y) by\ z = \cos(x. y)$. Show that;

$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2}$ (5 marks)

b) The base of radius r of a right circular cone is increasing at the rate of 1.5cm/s while the perpendicular height is decreasing at the rate of 0.8 cm/s. Find the rate at which the volume of the cone is changing at an instant when $r = 6.0cm$ and $h = 10.0\ cm$. (6 Marks)

c) Find and classify the stationary points of the surface $z = x^3 - 3x - xy^2$. (9 Marks)

QUESTION FOUR (20 Marks)

a) Given that $f(x)$ is a continuous function over the interval $[a, b]$ prove that.

$\frac{d}{dx}\int_a^x f(t)dt = f(x) \quad \forall\, x \in (a, b)$

(3 Marks)

b) Find

i) $\int 2x\sqrt{x^2 + 3}\ dx$. (3 Marks)

ii) $\int \left(\frac{19x-28}{x^2-5x+6}\right) dx$. (5 Marks)

iii) $\int (4x + 2) \sin 3x dx$. (3 Marks)

c) Find the length of the curve

$y = \frac{1}{6}x^3 + \frac{1}{2x}$

Between the points where $x = 1$ and $x = 3$. (6 Marks)

QUESTION FIVE (20 Marks)

a) Find the volume of the solid bounded by the planes $z = 0, y = 1, x = 0, x = 3$ and the surface $z = x^2 + xy$. (4 Marks)

b) By changing the order of integration, evaluate the double integral;

$I = \int_0^\pi \int_x^\pi \left(\frac{\sin y}{y}\right) dydx$

(4 Marks)

c) Sketch the domain of integration of the double integral;

$$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$$

Hence evaluate the double integral;

$$I = \int_0^1 \int_0^{\sqrt{1-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$$

(5 Marks)

d) Use the transformations $x = {}^{u}/_{v}$ and $y = u.v$ to find the area of a region bounded by the curves $xy = 1, xy = 9$ and the lines $y = x$ and $y = 4x$ in the first quadrant. (7 Marks)