MURANG'A UNIVERSITY COLLEGE

(A Constituent College of Jomo Kenyatta University of Agriculture & Technology)

FIRST YEAR/SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF HUMAN RESOURCE MANAGEMENT

HBC 2111: MANAGEMENT MATHEMATICS II

DATE: AUGUST TIME:

Instructions:-

- Answer question one and any other two questions

Question one

(a) Lake Naivasha Floataway Tours has Sh. 4 million that may be used to purchase new rental boats for hire during the coming December holidays. The boats can be purchased from the two different manufacturers. Pertinent data concerning the boats are summarized below:

Boat type	Manufacturer	Cost (Sh.)	Maximum Seating capacity	Expected Daily profit (Sh.)
Speedhawk	Sleekboat	60,000	3	7,000
Silverbird	Sleekboat	70,000	5	8,000
Chui	Racer	50,000	2	5,000
Simba	Racer	90,000	6	11,000

Floataway Tours like to purchase at least 50 boats and would like to purchase the same number from Sleekboat as from Racer to maintain goodwill. At the same time, Floataway Tours wishes to have a total seating capacity of at least 200

Required:

Formulate the above problem (5 marks)

b) HBC Ltd. manufactures and sells two interdependent products; Bora and Kizuri. The demand functions for the products are given by $P_1 = 800 – X – 2Y$ and $P_2 = 1,100 – X – 2.5Y$

Where: P_1 is the unit price of Bora and P_2 is the unit price of Kizuri

X and Y are the number of units sold for Bora and Kizuri respectively.

The total cost of producing both products is given by the function:

$TC = 150X + 50Y$

Required:

The number of units of each product required to maximize total profit. (8 marks)

c) The manufacturer of Tamu Soft drinks has been facing stiff competition on its main brand Tamu soda. The management is considering an extensive advertising and rebranding campaign for Tamu soda. After the advertising and rebranding campaign the transition matrix is expected to change as follows:

		To	
		Tamu	Others
From	Tamu	0.90	0.10
	Others	0.30	0.70

Required: The equilibrium state proportion of consumers using Tamu after the advertising campaign. (5 marks)

d) Solve for x, y and z using inverse method (8 marks)

$30X + 30Y + 20Z = 13000$

$30X + 20Y = 8500$

$10X + 30Y + 10Z = 6000$

e) A firm carries out a test launch of a new product. It estimates from this test that if it went into full-scale production it would sell between 1,000 and 2,500 units per week, and that its weekly revenue in (sh 000) over this range of sales could be represented by the equation:

$R = -x^2 + 5x$, Where x is the weekly output in thousands of units.

From experience the firm estimates that its marginal costs in thousands of shillings could be represented by the equation:

$MC = x^2 – x + 2$. Fixed costs will be Sh.500 per week. It is assumed that the entire output is sold.

Required: Determine the average cost and revenue equations for this firm. (4 marks)

Total 30 marks

Question two

Duplex Textile Ltd. makes executive and standard dresses. An executive dress requires 60 minutes for cutting, 50 minutes for sewing, 40 minutes for finishing and 15 minutes for inspection and packaging. A standard dress requires 42 minutes for cutting, 30 minutes for sewing, 60 minutes for finishing and 6 minutes for inspection and packaging.

The firm employs 35 tailors who spend a maximum of 630 hours for cutting, 600 hours for sewing, 708 hours for finishing and 135 hours of inspection and packaging every week. The profit contribution per dress is Sh.1,500 for the executive dress and Sh.1,350 for the standard dress.

Required:

(a) Formulate the linear programming model. (6 marks)

(b) Solve the linear programming model graphically. (6 marks)

(c) Calculate the slack or surplus values for each constraint and interpret their meaning. (4 marks)

(d) Calculate the dual price of an hour of finishing time. (4 marks)

(Total: 20 marks)

Question three

(a) Define the following terms as used in Markovian analysis:

(i) Transition matrix (2 marks)

(ii) Initial probability vector (1 marks)

(iii) Equilibrium (1 marks)

(iv) Absorbing state (2 marks)

(b) A company employs four classes of machine operators (A, B, C and D); all new employees are hired as class D and, through a system of promotion, may work up to a higher class. Currently, there are 200 class D, 150 class C, 90 class B and 60 class A employees. The company has signed an agreement with the union specifying that 20 percent of all employees in each class be promoted, one class in each year. Statistics show that each year 25 percent of the class D employees are separated from the company by reasons such as retirement, resignation and death. Similarly 15 percent of class C, 10 percent of class B and 5 percent of class A employees are also separated. For each employee lost, the company hires a new class D employee.

Required:

(i) The transition matrix (5 marks)

(ii) The number of employees in each class two years after the agreement with the union. (9 marks)

(Total: 20 marks)

Question four

Gatheru and Kabiru Certified Public Accountants have recently started to give business advice to their clients. Acting as consultants, they estimated the demand curve of a client's firm to be:

$AR = 200 - 8Q$

Where AR is average revenue in millions of shillings and Q is the output in units.

Investigations of the client firm's cost profile shows that marginal cost (MC) is given by:

$MC = Q^2 - 28Q + 211$ (in millions of shillings)

Further investigations have shown that the firm's cost when not producing output is Sh.10 million.

Required:

(i) The equation of total cost (3 marks)

(ii) The equation of total revenue (2 marks)

(iii) An expression for profit (2 marks)

(iv) The level of output that maximizes profit (5 marks)

(v) The equation of marginal revenue (2 marks)

b) Critique the assumptions of markov analysis 6 marks Total 20 marks

Question five

(a) The Young Children's Fund (YCF) is planning its annual fund-raising campaign for its December school holiday camp for disadvantaged children. Campaign expenditures will be incurred at a rate of Sh. 10,000 per day. From past experience, it is known that contributions will be high during the early stages of the campaign and will tend to fall off as the campaign continues. The function describing the rate at which contributions are received is:

$C(t) = -100t^2 + 200,000$

Where t = days of the campaign

$C(t)$ = rate at which contributions are received in shillings per day.

The fund wants to maximize the net proceeds from the campaign.

Required:

(i) The number of days the campaign should be conducted to maximize the net proceeds. (3 marks)

(ii) The total campaign expenditure. (2 marks)

(iii) The total contributions expected to be collected. (5 marks)

(iv) Net proceeds from the campaign. (1 mark)

(b) The national office of a car rental company is planning its maintenance for the year. The company's management are interested in determining the company's needs for certain repair parts. The company rents saloon cars, station wagons and double cab pick-ups. The matrix N shown below indicates the number of each type of vehicle available for renting in the four regions of the county.

	Saloons	Station Wagons	Double cabs	
N =	160	400	500	Cost
	150	300	200	Central
	100	100	150	Western
	120	400	300	Highlands

Four repair parts of particular interest, because of their cost and frequency of replacement, are fan belts, spark plugs, batteries and tyres. On the basis of studies of maintenance records in different parts of the country, the management has determined the average number of repair parts needed per car during a year.

These are summarised in matrix R below:

	Saloons	Station Wagons	Double cabs	
R =	17	16	15	Fan belts
	12	8	5	Plugs
	9	7	5	Butteries
	4	7	6	Tyres

Required:

(i) The total demand for each type of car. (3 marks)

(ii) The total number of each repair part required for the fleet. (3 marks)

(iii) If matrix C below contains the cost per unit in shillings for fan belts, spark plugs, batteries and tyres, calculate the total costs for all parts.

C = (1250, 800, 6500, 8000) (3 marks) Total 20 marks

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Question one

(a) Let: X_1 = The number of speed hawk boats ordered
X_2 = The number of silverbird boats ordered
X_3 = The number of chui boats ordered
X_4 = The number of Simba boats ordered.

Objective function: Maximise $Z = 7000X_1 + 8000X_2 + 5000X_3 + 11000X_4$ subject to:

1. $60{,}000X_1 + 70{,}000X_2 + 50{,}000X_3 + 90{,}000X_4 \leq 4{,}000{,}000$ (Budgeted amount)
2. $X_1 + X_2 + X_3 + X_4 \geq 50$ (Least number of boats to purchase)
3. $X_1 + X_2 = X_3 + X_4$ (Equal number of boats to be purchased from the two manufacturers)
4. $3X_1 + 5X_2 + 2X_3 + 6X_4 \geq 200$ (Least capacity)
5. $X_1, X_2, X_3, X_4 \geq 0$ (Non-negativity)

b) Revenue for Bora $R_1 = P_1X = 800X - X^2 - 2xy$
Revenue for Kizuri, $R_2 = P_2y = 1100y - xy - 2.5y^2$
Total revenue, $R = R_1 + R_2 = 800x + 1100y - x^2 - 3xy - 2.5y^2$
Total cost, $C = 150x + 50y$
Profit, $\pi = R - C = 650x + 1050y - x^2 - 3xy - 2.5y^2$

F.O.C: $\frac{d\pi}{dx} = 650 - 2x - 3y = 0 \quad \Leftrightarrow \quad 650 = 2x + 3y$ ….. (i)

$\frac{d\pi}{dy} = 1050 - 3x - 5y = 0 \quad \Leftrightarrow \quad 1050 = 3x + 5y$ ….. (ii)

3x(i) $\Leftrightarrow$ $1950 = 6x + 9y$
2x(ii) $\Leftrightarrow$ $\underline{2100 = 6x + 10y}$
$150 = y$

$$x = \frac{650 - 3(150)}{2} = 100$$

S.O.C: $\frac{d^2\pi}{dx^2} = -2$, $\frac{d^2\pi}{dx^2} = -5$ $\Big\} < 0$ => Profit is maximum when Bora sells 100 units and Kizuri sells 150 units.

c) After the advertising campaign:
Let: q_1 = Equilibrium state proportion of consumers of Tamu-Cola.

$q_2 = 1-q_1$ = Equilibrium state proportion of consumers of others.

$$\begin{pmatrix} q_1 & 1-q_1 \end{pmatrix}\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} = \begin{pmatrix} q_1 & 1-q_1 \end{pmatrix}$$

$0.9q_1 + 0.3 - 0.3q_1 = q_1$

$-0.4q_1 = -0.3$

$q_1 = 0.75 = 75\%$

$q_2 = 1-0.75 = 0.25 = 25\%$.

d) Using inverse method:

AB=C i.e. $B=CA^{-1}$

If $A = \begin{pmatrix} 30 & 30 & 20 \\ 30 & 20 & 0 \\ 10 & 30 & 10 \end{pmatrix}$

Then,The minor element matrix $M = \begin{pmatrix} 200 & 300 & 700 \\ -300 & 100 & 600 \\ -400 & -600 & -300 \end{pmatrix}$

The cofactor matrix $C = \begin{pmatrix} 200 & -300 & 700 \\ 300 & 100 & -600 \\ -400 & 600 & -300 \end{pmatrix}$

Adjoint matrix= $\begin{pmatrix} 200 & 300 & -400 \\ -300 & 100 & 600 \\ 700 & -600 & -300 \end{pmatrix}$

Therefore $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = 1/11000 \text{ x} \text{Adjoint x} \begin{pmatrix} 13000 \\ 8500 \\ 6000 \end{pmatrix}$

$$= \begin{pmatrix} 250 \\ 50 \\ 200 \end{pmatrix}$$

e) TC = $\int mc = \int (x^2 - x + 2)dx$

TC = $\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + f$

TC = $\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + 0.5$

Average cost = $\frac{\text{Total cost}}{\text{Output}} = \frac{TC}{X}$

A.C = $\frac{1}{3}x^2 - \frac{1}{2}x + x + \frac{0.5}{x}$

R = $-x^2 + 5x$

Average revenue = $\frac{\text{Total Revenue}}{\text{Output}} = \frac{R}{x}$

AR = $-x + 5$

Question two

(a) Let: X_1 = Number of executive dresses made and sold.
X_2 = Number of standard dresses made and sold.
Z = Total contribution.

Max Z = $1500X_1 + 1350X_2$

S.t

1. $60X_1 + 42X_2 \leq 37800$ (cutting time)
2. $50X_1 + 30X_2 \leq 36000$ (sewing time)
3. $40X_1 + 60X_2 \leq 42480$ (finishing time)
4. $15X_1 + 6X_2 \leq 8,100$ (inspection and packaging time)
5. $X_1, X_2 \geq 0$ (Non-negativity)

(b) Boundary Lines

1. $60X_1 + 42X_2 \leq 37800$

X_1	0	630
X_2	900	0

2. $50X_1 + 30X_2 \leq 36000$

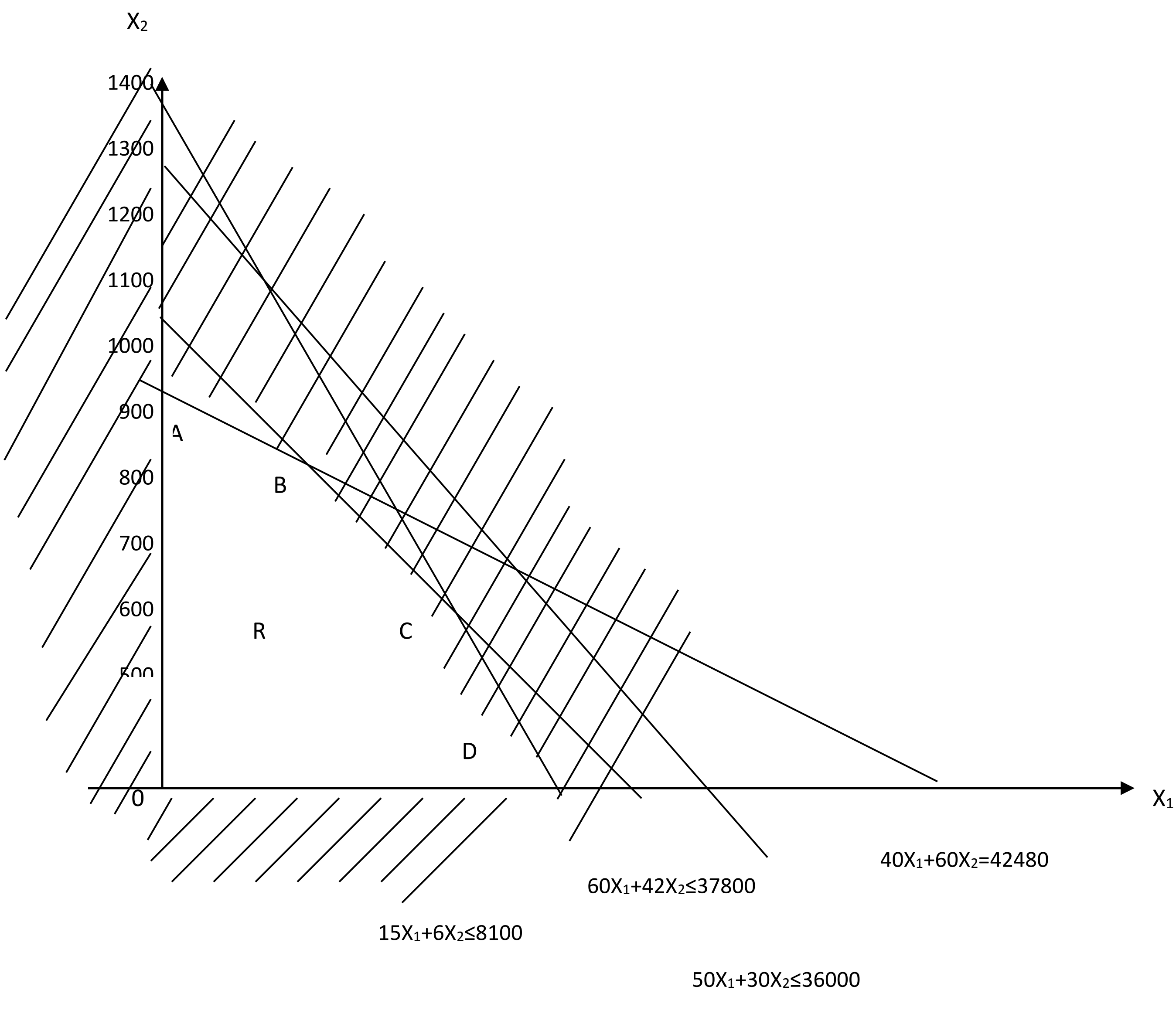
X_1	0	720
X_2	1200	0

3. $40X_1 + 60X_2 \leq 42480$

X_1	0	1062
X_2	708	0

4. $15X_1 + 6X_2 \leq 8100$

X_1	0	540
X_2	1350	0



Optimality check using corner point method.

Point	(X_1,X_2)	Total contribution, $Z = 1500X_1 + 1350X_2$
A	(0, 708)	Sh.955,800
B	(252, 540)	Sh.1,107,000 * Maximum
C	(420, 300)	Sh.1,035,000
D	(540, 0)	Sh.810,000

Optimal solution: Make 252 executive dresses and 540 standard dresses.
Maximum contribution = Sh.1,107,000.

(c) Slacks:

$S_1 = 37800 – \{60(252) + 42(540)\} = 0$

$S_2 = 36000 – \{40(252) + 60(540)\} = 7200$ minutes = 120 hours

$S_3 = 42480 – 40(252) – 60(540) = 0$

$S_4 = 8100 – 15(252) – 6(540) = 1080$ minutes
= 18 hours

Constraint 2 has 120 hours of sewing unused.
Constraint 4 has 18 hours of inspection unused.

(d) dual price: Assume an extra hour of finishing was donated and used. Therefore

$60X_1 + 42X_2 \leq 37800$(cutting time)

$40X_1 + 60X_2 \leq 42540$ (finishing time)

Solving for X_1=250.6785

X_2=541.875

Z=1500(250.6785)+1350(541.875)=1107562.5

Dual price: 1107562.5-1107000=sh 562.5

Question three

a) (i) Transition matrix is a matrix whose elements are probabilities that a process will change from one state to another state in a defined period of time.

(ii) Initial probability vector is a vector that gives the initial (starting) probabilities of each state. When initial probability vector is multiplied by the transition matrix we get future or predicted probability vector.

(iii) Equilibrium state is the longterm or steady state of a markov process. Provided the assumptions of markov process persist, the system finally reaches an equilibrium called steady or long term status i.e a state where no further net change occurs. At equilibrium the following holds.
(Equilibrium state vector) (Transition matrix) = (Equilibrium state vector)

(iv) An absorbing state is a state which cannot be left once it has been entered.

(b) (i) Transition matrix,

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.95 & 0 & 0 & 0.05 \\ 0.20 & 0.70 & 0 & 0.10 \\ 0 & 0.20 & 0.65 & 0.15 \\ 0 & 0 & 0.20 & 0.80 \end{bmatrix} \end{array}$$

(ii) - Number of employees in each class after one year:

$$\begin{array}{cccc} A & B & C & D \end{array}$$

$$\begin{pmatrix} 60 & 90 & 150 & 200 \end{pmatrix} \begin{bmatrix} 0.95 & 0.00 & 0.00 & 0.05 \\ 0.20 & 0.70 & 0.00 & 0.10 \\ 0.00 & 0.20 & 0.65 & 0.15 \\ 0.00 & 0.00 & 0.20 & 0.80 \end{bmatrix}$$

$$\begin{array}{cccc} A & B & C & D \end{array}$$

$$= \begin{pmatrix} 75 & 93 & 137.5 & 194.5 \end{pmatrix}$$

- Number of employees in each class after two years:

$$\begin{pmatrix} 75 & 93 & 137.5 & 194.5 \end{pmatrix} \begin{bmatrix} 0.95 & 0.00 & 0.00 & 0.05 \\ 0.20 & 0.70 & 0.00 & 0.10 \\ 0.00 & 0.20 & 0.65 & 0.15 \\ 0.00 & 0.00 & 0.20 & 0.80 \end{bmatrix}$$

$$\begin{array}{cccc} A & B & C & D \end{array}$$

$$= \begin{pmatrix} 89.85 & 92.6 & 128.275 & 189.275 \end{pmatrix}$$

Class	Number of Employees
A	90
B	93
C	128
D	189
Total	500

Question four

a) (i) TC = $\int MCdQ = \int (Q^2 - 28Q + 211)dQ$

$TC = \frac{1}{3}Q^3 - 14Q^2 + 211Q + f$

$TC = \frac{1}{3}Q^3 - 14Q^2 + 211Q + 10$

(ii) AR $= \frac{TR}{Q}$

TR = AR x Q
= (200 – 8Q)xQ

TR = $200Q - 8Q^2$

(iii) Profit = Total revenue – Total cost

Π = TR – TC
$= 200Q - 8Q^2 - \{\frac{1}{3}Q^3 - 14Q^2 + 211Q + 10\}$
$\Pi = -\frac{1}{3}Q^3 + 6Q^2 - 11Q - 10$

(iv) For maximum or minimum, $\frac{d\pi}{dQ} = 0$

$$\frac{d\pi}{dQ} = -Q^2 + 12Q - 11 = 0$$

$$Q^2 - 12Q + 11 = 0$$

$$Q = \frac{12 \pm \sqrt{144 - 4x1x11}}{2x1} = \frac{12 \pm 10}{2}$$

Q = 11 or Q = 1

Second order condition, $\frac{d^2\pi}{dQ^2} = -2Q + 12$

When Q = 11, $\frac{d^2\pi}{dQ^2} = -2(11) + 12 = -10 < 0$ => Max. turning point

When Q = 1, $\frac{d^2\pi}{dQ^2} = -2(1) + 12 = 10 > 0$ => Min. turning point

$\therefore$ Level of output that maximizes profit is 11 units.

(v) MR = $\frac{d(TR)}{dQ}$ = 200 – 16Q

b)

Question five

(a) (i) At the intersection of expenditure and contribution rate functions:

$$-100t^2 + 200,000 = 10,000$$

$$\frac{-100t^2}{-100} = \frac{-190,000}{-100}$$

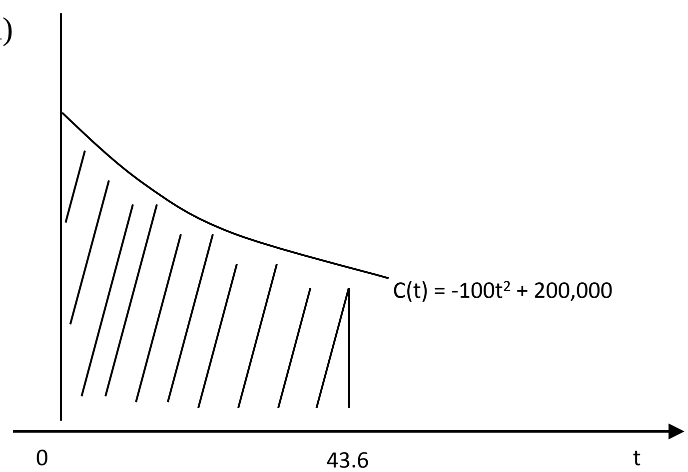
$$t^2 = 1,900$$

$$t = 43.6 \text{ days}$$

(ii) Total campaign expenditure = Sh.10,000 x 43.6

= Sh.436,000

(iii)



Total contribution

= Area under the curve between t = 0 and t = 43.6 days

= $\int_0^{43.6} (-100t^2 + 200,000)dt$

$$= \left[\frac{-100}{3} t^3 + 200{,}000t \right]_0^{43.6}$$

$$= \text{Sh},5{,}957{,}271$$

(iv) Net proceeds = 5,957,271 – 436,000 = Sh.5,521,271

(b) (i) Total Demand

$$= \begin{bmatrix} 160+150+100+120 \\ 400+300+100+400 \\ 500+200+150+300 \end{bmatrix} = \begin{bmatrix} 530 \\ 1200 \\ 1150 \end{bmatrix} \begin{matrix} \text{Saloons} \\ \\ \text{Station Wagons} \end{matrix}$$

(ii) Total number of repair part required for the fleet:

$$= \begin{bmatrix} 17 & 16 & 15 \\ 12 & 8 & 5 \\ 9 & 7 & 5 \\ 4 & 7 & 6 \end{bmatrix} \begin{bmatrix} 530 \\ 1200 \\ 1150 \end{bmatrix}$$

$$= \begin{bmatrix} 45{,}460\, fanbelts \\ 21{,}710\, plugs \\ 18{,}920 butteries \\ 17{,}420 tyres \end{bmatrix}$$

(iii) Total cost

$$= \begin{pmatrix} 1250 & 800 & 6500 & 8000 \end{pmatrix} \begin{pmatrix} 45{,}460 \\ 21{,}710 \\ 18{,}920 \\ 17{,}420 \end{pmatrix}$$

= Sh.336,533,000