MURANG'A UNIVERSITY COLLEGE (MRUC)

A constituent College of Jomo Kenyatta University of Agriculture and Technology (JKUAT)

HBC 2103

MARKING SCHEME (FOR SPECIAL / SUPPLEMENTARY EXAM

1 a. i) A, B be sets. If all elements of A are found is B than A is a subset of B.

$A \underline{C} B.$ ①

ii. Let X be a set and ξ its universal set such that $X \underline{C} \xi$. Compliments of X denoted by X^c is

$X^c = \{ a : a \in \xi \text{ and } a \notin X\}$ ①

iii. The universal set is the set containing all elements in a topic under discussion. ①

iv. Venn diagram is a pictorial representation of a set. ①

b. u = N = {1,2,3,……. Y, A= {1,2,3,4}, B = {3,4,5,6,7}

i) {AUB}

= {1,2,3,4} U {3,4,5,6,7}

= {1,2,3,4,5,6,7} ①

ii) AΠB = {1,2,3,4} Π { 3,4,5,5,7}
= {3,4} ①

iii. $A^c = \{ x : x \in N \quad \text{and } x \notin A\}$

$A^c = \{ 5,6,7, \ldots\ldots\}.$ ①

Similarly
iv. $B^c = \{ 1,2,8,9,10, \ldots\ldots\ldots\}$ ①

c) $f(x) = \dfrac{x^4 + x^2 - x^2 + 1}{x^2-1}$

$= \dfrac{x^2(x^2 + 1) - 1(x^2 + 1)}{X^2 - 1}$ ①

$= (x^2 + 1)$ ①

$= x + 1 = l(x)$ ①

$\underset{x \to +1}{\text{Lin}} f(x) = \underset{x \to +1}{\text{Lin}} \dfrac{x^4 - x^2 - x^2 + 1}{x^2 - 1} = \underset{x \to +1}{\text{Lin}} x^2 + 1$ ①

$= (1)^2 + 1$ ①

$= 2.$

d) $x - 1 \overline{\big| x^3 + 4x^2 + x - 6}$ with quotient $x^2 + 5x + 6$ ①

$\underline{x^3 - x^2}$

$5x^2 + x - 6$

$\underline{5x^2 - 5x}$

$6x - 6$ ①

$\underline{6x - 6}$

Also x2 + 5x + 6

$= x^2 + 2x + 3x + 6$

$= x(x + 2) + 3(x + 2)$

$(x + 2)(x + 3)$ ①

∴ $x^3 + 4x^2 + x - 6 = (x- 1)(x +2)(x +3)$

∴ If $x^3 + 4x^2 + x - 6 =0$

⇨ $(x -1)(x + 2)(x + 3) = 0$

$X = 1, x = -2, x = -3$ ①

e) Initial investment, α=ƒ100

After 1 year $100 \times \frac{108}{100}$ = ƒ108

After 2 years 108 x 108 = £116.64

Common ratio, r of 100 + 108 + 116.64 + -----

r =1.08

i. after 10 years $U_n = \alpha r^{1-1}$

$u_{10} = 100(1.08)^9$ ①

u_{10} = £199.900 ①

≃ £ 199.9

≃ £ 200 to 3 s.f ①

ii. $U_n = 300$

$300 = 100(1.08)^{n-1}$ ①

$n - 1 = \frac{\log 3}{\log 1.08}$ ①

$n - 1 = 14.27$

$n = 15.27$ ①

f) $y = 2x3 - 9x2 + 12x + 5$

$\frac{dy}{dx} = 6x2 - 18x + 12$ ①

$= 6x2 - 18x + 12.$

$^{dy}/_{dx}|$= b(б) + 18(∂)+ 12 ①

= 12 ①

Nature ƒ turning point

$\frac{d^2y}{dx^2} = 12x - 18$

$\left.\frac{d^2y}{dx_2}\right|_{12} = 12(12) - 18$ ①

= + 126

∴ A minimum Turning point

$S = 3t^3 - 20t^2 + 40t$

2a) i) $S (+) = 3(3)^3 -20 (3)^2 + 40(3)$

$= 81 - 180 + 120$

$=21$ metres

ii. $v(t) = \frac{ds}{dt}$

$= \frac{d}{dt} (3t^3 - 20t^2 + 40t)$

$v(t) = 9t^{L -}40t + 40$

iii. 9t2 – 40t + 40 = 0

$t = \frac{-b + \sqrt{b^2-4ac}}{2G}$, $\alpha =9, b = -40, c= 40$

$t = \frac{-(-40) + \sqrt{(-40)^2 - (4) (9) (40}}{2(9)}$

$= \frac{40 + \sqrt{1600 - 1440}}{18}$

$= \frac{40 + \sqrt{160}}{18}$

$t = \frac{40 + 12.6491}{18}$

$t = \frac{40 + 12.6491}{18} = \underline{\underline{2.92495S}}$ $\underline{\underline{or}}$ $t = \frac{40 - 12.6491}{18} = \underline{\underline{1.51949S}}$

iv) when + = 0, initial velocity is

$v(0) = 9(0) - 40 (0) + 40$

$= 40$ m/s

v) acceleration $= \frac{dv}{dt}$

$\alpha= \underline{d} (9t^2 - 40t +40)$

$\alpha= 18t - 40$

vi) $\alpha = 18t - 40$ $\alpha+$, $t = 0$

α = Intial acceleration $= 18(0) -40$

$= -40 m/s^2$.

b) i. $\int_0^1 (3x + 1)^5 dx$.

Let $(3x + 1) = t$ ①

$\frac{dt}{dx} = 3$

$\frac{1}{3} dt = dx$

$\therefore \int_0^1 (3x + 1) dx^5 = \int_0^1 (t)^5 \frac{1}{3} dt$. ①

$= \left[\frac{t^6}{6} \right]_0^1$ replacing $t = 3x + 1$

$= \left[\frac{(3x + 1)^6}{6} \right]$ ①

$= 3\frac{(1) + 1}{6} - 3\frac{(0) + 1}{6}$ ①

$= \frac{4}{6} - \frac{1}{6}$

$= \frac{3}{6} = \frac{1}{2}$ ①

4. a) Let α be the first term and r be the rate of interest.

α=£ 250.

r =6%.

End of 1st year $\frac{106}{100} \times 250$ = £ 265 ①

End of 2nd year $\frac{106}{100} \times 265$ = £ 280.9 ①

End of 3rd year $\frac{106}{100} \times 280.9$ = £297.754 ①

Series required is 265 + 280.9 + 297.754 + ----①
with α = 265 and r = 1.06.

i. $T_n = \alpha r^{n-1}$

$T_{15} = 265 (1.06)^{14}$ ①

= £ 599.1395483

$\simeq$ £ 599.14 ①

ii. nth value = £ 750

$\Rightarrow 750 = \alpha r^{n-1}$

$750 = 265\,(1.06)^{n-1}$

$(1.06)^{n-1} = \frac{750}{265} = 2.830188679$

$(n-1) \log 1.06 = \text{Log } 2.8302.$

$n-1 = \frac{\log 2.8302}{\log 1.06} = \frac{0.4518}{0.0253} = 17.86$

$= 17.86$

$n = 17.86 + 1 = 18.86$

$n \simeq 19$ years

b) 10th term of A series $= \alpha + \alpha d$

5th term of A series $= \alpha + 5d$

Since 10th term = twice 5th term.

$\Rightarrow \alpha + \alpha d = 2(\alpha + 5d)$

$\alpha + 9d = 0$ ______ (i)

Also 30th term = 60

$\Rightarrow \alpha + 2\alpha d = 60$ ______ (ii)

from (i) $a = -9d.$

$\therefore$ in (ii)

$-9d + 29d = 60$

$20d = 60$

$d = 3$

Since $\alpha = 9d.$

$\alpha = -9(3)$

$\alpha = -27$

$S_n = \frac{n}{2}(2a + (n-1)\,d$

$S_q = \frac{9}{2}(2(-27) + (9-1)3) = \frac{9}{2}(-54 + 24) = -135$

$S_{20} = \frac{20}{2}(2(-27) + (20-1)3) = 10(-54 + 57) = 30$

The sum from 9th term to 20th term is

$S_{20} - s_9 = 30 - (-135)$

$= 165$

C i.) $ƒ(x)= 2x -1$ $g(x) = x^2 + 5x -2.$

$g(f(x) = g(2x -1)$

$= g(2x-1)$

$= (2x -1)^2 -5(2x – 1) -2$ ①

$= 4x^2 – 4x+1 +10x – 5 -2$ ①

$= 4x^2 +6x +6$ ①

ii) g of $(2) = 4(2)^2 +6(2) + 6$ ①

$= 16 + 12 + 6$

$= 34$ ①

3. Matrix Eqn. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 33 \\ 2 \end{pmatrix}$ ①

$X= \frac{Dx}{D}$ $y = \frac{Dy}{D}$ $Z=\frac{D2}{D}$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & -2 \end{vmatrix} = (6+8) – (-4 -2) + (-4 +9) = 35$ ①

$Dx = \begin{vmatrix} 4 & 1 & 1 \\ 33 & -3 & 4 \\ 2 & -2 & -2 \end{vmatrix} = 4(6 + 8) – (-66 - 8) + (-66 + 8) = 70$

$x = \frac{70}{35} = 2$ ①

$Dy = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 33 & 4 \\ 3 & 2 & -2 \end{vmatrix} = (-66 – 8) -4 (-4 -12) + (4 -99) = -105$ ①

$y = -\frac{105}{35} = 3$ ①

$= (-6 + 66) – (4-99) + 4(-4 + 9)$

$= 175$ ①, $z= \frac{175}{35} = 5$ ①

b) $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1- & -1 \\ 3 & 2 & 1 \end{pmatrix}$

Co – factors $A_{11} = + (-1 + 2) = 1$ $A_{12} = -(2 + 3) = -5$

$A_{21} = - (2 + 6) = -8$ $A_{22} = + (1 + 9) = 10$

$A_{31} = + (-2 -3) = -5$ $A_{32} = -(-1 +6) = -5$

$A_{13} = + (4 +3) = 7$

$A_{23} = -(2 -6) = 4$

$A_{33} = + (-1-4) = -5$

$\therefore$ matrix of cofactors is $\begin{pmatrix} 1 & -5 & 7 \\ -8 & 10 & 4 \\ -5 & -5 & -5 \end{pmatrix}$

Ad joints A, Adj A = $\begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix}$

$A^{-1} = \frac{1}{141}$ Adj A where (A) = det A = 1(-1 + 2) – 2(2+3) – (4 + 3)
= 1 – 10 – 21
=-30

$\therefore A^{-1} = -\frac{1}{30}\begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix}$

For the system
$x_1 + 2x_2 - 3x_3 = 3$
$2x_1 - x_2 - x_3 = 11$
$3x_1 + 2x_2 + x_3 = -5$

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 4^{-1} \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = {}^{-1}/_{30} \begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix} \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = {}^{-1}/_{30} \begin{bmatrix} -60 \\ 120 \\ 90 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$$

$x_1 = 2$, $x_2 = -4$, $x_3 = -2$

5. a) i. factors of f(x) are: (x+3), (x -2) and (x-4) ①
f(x) = k (x+3) (x-2) (x-4) ①
f(o) = k(3) (-2) (-4) = -12
$\Rightarrow$ 24k = -12
K = $-{}^{1}/_{2}$ ①

Hence f(x) = -1/2 (x+3) (x-2) (x-4) ①

ii.
$$\begin{array}{r l l}
 & \underline{\;3x2 - 4x - 1 \qquad\qquad} & \\
x^2+2 \big) & 3x^4 \; 4x^3 + 5x^2 \,{}^{-} 7x + 4 & \\
 & \underline{3x^4 \qquad +6x2 \qquad\qquad} & \\
 & \qquad -4x^3 - x^2 + 7x + 4 & \\
 & \qquad \underline{-4x^3 \qquad -8x} & \text{①} \\
 & \qquad\quad -x2 + x + 4 & \\
 & \qquad\quad \underline{-x2 \quad -2} & \\
 & \qquad\quad\; X + 6 & \text{①}
\end{array}$$

Hence

$3x^4 - 4x^3 + 5x^2 - 7x + 4 = (3x^2 - 4x - 1)\,(x^2 + 2) + x + 6$ ①

b. $\dfrac{x^{-2/3} y^{1}/_{2} \, Z^{1/3}}{(x^{2/3}\, y^2\, Z^{1/3})\frac{1}{3}}$ $= \dfrac{x^{-2/3}\, y^{1/2}\, Z^{1/3}}{x^{2}/_{9}\, y_{2/3}\, Z_{1/9}}$

$= X^{-2}/_{3}\,{}^{-2}/_{9}\; y^{1}/_{2}{}^{-2}/_{3}\; Z^{1}/_{3}\; -{}^{1}/_{9}$ ①

$= x^{-8}/_{9}\;\; y^{-1}/_{6}\;\; z^{2}/_{9}$ ①

$= Z_{2/9}$

$X^{8}/_{9}\; Y^{-1}/_{6}$ ①

b) $a^2\, b^{1}/_{2}\, c^{1}/_{4}\;\; a^{1}/_{4}\, b^{1/4} \;\; =$