# MURANG'A UNIVERSITY COLLEGE (MRUC)

A constituent College of Jomo Kenyatta University of Agriculture and Technology (JKUAT)

**HBC 2103**

# MARKING SCHEME (FOR SPECIAL / SUPPLEMENTARY EXAM)

1 a. i) Let A, B be sets. If all elements of A are found in B than A is a subset of B.

$A \underline{C} B$. ①

ii. Let X be a set and ξ its universal set such that $X \underline{C} \xi$. Compliments of X denoted by $X^c$ is

$X^c = \{ a: a \in \xi \text{ and } a \notin X\}$ ①

iii. The universal set is the set containing all elements in a topic under discussion. ①

iv. Venn diagram is a pictorial representation of a set. ①

b. u = N = {1,2,3,……. }, A= {1,2,3,4}, B = {3,4,5,6,7}

i) {AUB}

= {1,2,3,4} U {3,4,5,6,7}

= {1,2,3,4,5,6,7} ①

ii) AΠB = {1,2,3,4} Π { 3,4,5,5,7}

= {3,4} ①

iii. $A^c = \{ x : x \in N$ and $x \notin A\}$

$A^c = \{ 5,6,7, \ldots\ldots\}$. ①

Similarly

iv. $B^c = \{ 1,2,8,9,10, \ldots\ldots\ldots\}$ ①

c) $f(x) = \frac{x^4 + x^2 - x^2 + 1}{x^2 - 1}$

$= \frac{x^2(x^2 + 1) - 1(x^2 + 1)}{X^2 - 1}$ ①

$= (x^2 + 1)$ ①

$= x + 1 = l(x)$ ①

$\underset{x \to +1}{\text{Lin}} f(x) = \underset{x \to +1}{\text{Lin}} \frac{x^4 - x^2 - x^2 + 1}{x^2 - 1} = \underset{x \to +1}{\text{Lin}} x^2 + 1$ ①

$= (1)^2 + 1$ ①

$= 2.$

d)
$$\begin{array}{r|l} & \underline{x^2 + 5x + 6} \\ x - 1 & x^3 + 4x^2 + x - 6 \\ & \underline{x^3 - x^2} \\ & 5x^2 + x - 6 \\ & \underline{5x^2 - 5x} \\ & 6x - 6 \\ & \underline{6x - 6} \end{array}$$

① ①

Also $x^2 + 5x + 6$

$= x^2 + 2x + 3x +$

$= x(x + 2) + 3(x + 2)$

$(x + 2)(x + 3)$ ①

$\therefore$ $x^3 + 4x^2 + x - 6 = (x-1)(x+2)(x+3)$

$\therefore$ If $x^3 + 4x^2 + x - 6 = 0$

$\Rightarrow (x-1)(x+2)(x+3) = 0$

$X = 1, x = -2, x = -3$ ①

e) Initial Investment, $\alpha = ƒ100$

After 1 year $100 \times \frac{108}{100} = ƒ108$

After 2 years $\frac{108}{100} \times 108 = £116.64$

Common ratio, r of $100 + 108 + 116.64 + \text{-----}$

$r = 1.08$

i. after 10 years $U_n = \alpha r^{1-1}$

$u_{10} = 100(1.08)^9$ ①

$u_{10} = £199.900$ ①

$\simeq £\ 199.9$

$\simeq £\ 200$ to 3 s.f ①

ii. $U_n = 300$

$300 = 100(1.08)^{n-1}$ ①

$n - 1 = \frac{\log 3}{\log 1.08}$ ①

$n - 1 = 14.27$

$n = 15.27$ ①

f) $y = 2x^3 - 9x^2 + 12x + 5$

$\frac{dy}{dx} = 6x^2 - 18x + 12$ ①

$= 6x^2 - 18x + 12.$

$\left.{}^{dy}/_{dx}\right| = b(\text{б}) + 18(\partial) + 12$ ①

$= 12$ ①

Nature ƒ turning point

$\frac{d^2y}{dx^2} = 12x - 18$

$\left.\frac{d^2y}{dx_2}\right|_{12} = 12(12) - 18$ ①

$= +126$

$\therefore$ A minimum Turning point

$S = 3t^3 – 20t^2 + 40t$

2a) i) $S(t) = 3(3)^3 - 20(3)^2 + 40(3)$ ①

$= 81 – 180 + 120$

$= 21$ metres

ii. $v(t) = \frac{ds}{dt}$

$= \frac{d}{dt}(3t^3 – 20t^2 + 40t)$

$v(t) = 9t^{L\,-}40t + 40$

iii. $9t^2 – 40t + 40 = 0$

$t = \frac{-b + \sqrt{b^2 - 4ac}}{2G}$ , α =9, b = -40, c= 40

$t = \frac{-(-40) + \sqrt{(-40)^2 – (4)(9)(40}}{2(9)}$

$= \frac{40 + \sqrt{1600 – 1440}}{18}$

$= \frac{40 + \sqrt{160}}{18}$

$t = \frac{40 + 12.6491}{18}$

$t = \frac{40 + 12.6491}{18} = \underline{\underline{2.92495S}}$ $\underline{\underline{\text{or}}}$ $t = \frac{40 – 12.6491}{18} = \underline{\underline{1.51949S}}$

iv) When + = 0, initial velocity is

$v(0) = 9(0) – 40(0) + 40$

$= 40$ m/s

v) Acceleration $= \frac{dv}{dt}$

$\alpha = \underline{d}(9t^2 – 40t + 40)$

$\alpha = 18t – 40$

vi) $\alpha = 18t – 40$ $\alpha +$, $t = 0$

α = Initial acceleration $= 18(0) - 40$

$= -40m/s^2$.

b) i. $\int_0^1 (3x + 1)^5 dx.$

Let $(3x + 1) = t$ ①

$\frac{dt}{dx} = 3$

$⅓\, dt = dx$

.

. . $\int_0^1 (3x + 1)^5\, dx = \int_0^1 (t)^5\, ⅓ dt.$ ①

$= \left[ \frac{t^6}{6} \right]_0^1$ replacing $t = 3x + 1$

$= \left[ \frac{(3x + 1)^6}{6} \right]_0^1$ ①

$= 3\frac{(1) + 1}{6} - 3\frac{(0) + 1}{6}$ ①

$= {}^4/_6 - {}^1/_6$

$= {}^3/_6 = {}^1/_2$ ①

4. a) Let α be the first term and r be the rate of interest.

α=£ 250.

r =6%.

End of 1st year $\frac{106}{100} \times 250 = £\ 265$ ①

End of 2nd year $\frac{106}{100} \times 265 = £\ 280.9$ ①

End of 3rd year $\frac{106}{100} \times 280.9 = £297.754$ ①

Series required is 265 + 280.9 + 297.754 + ----①

with α = 265 and r = 1.06.

i. $Tn = \alpha r^{n-1}$

$T_{15} = 265\ (1.06)^{14}$ ①

$= £\ 599.1395483$

$\simeq £\ 599.14$ ①

ii. nth value = £ 750

$\Rightarrow 750 = \alpha r^{n-1}$

$750 = 265\,(1.06)^{n-1}$

$(1.06)^{n-1} = \frac{750}{265} = 2.830188679$

$(n-1) \log 1.06 = \text{Log } 2.8302.$

$$n-1 = \frac{\log 2.8302}{\log 1.06} = \frac{0.4518}{0.0253} = 17.86 = 17.86$$

$n = 17.86 + 1 = 18.86$

$n \simeq 19$ years

b) 10th term of A series $= \alpha + 9d$

5th term of A series $= \alpha + 5d$

Since 10th term = twice 5th term.

$\Rightarrow \alpha + 9d = 2(\alpha + 5d)$

$\alpha + 9d = 0$ ——— (i)

Also 30th term = 60

$\Rightarrow \alpha + 29d = 60$ ———(ii)

from (i) $a = -9d.$

$\therefore$ in (ii)

$-9d + 29d = 60$

$20d = 60$

$d = 3$

Since $\alpha = 9d.$

$\alpha = -9(3)$

$\alpha = -27$

$S_n = \frac{n}{2}(2a + (n-1)\,d$

$S_q = \frac{9}{2}\,(2(-27) + (9-1)3) = \frac{9}{2}(-54 + 24) = -135$

$S_{20} = \frac{20}{2}\,(2(-27) + (20-1)3) = 10(-54 + 57) = 30$

The sum from 9th term to 20th term is

$S_{20} - s_9 = 30 - (-135)$

$= \underline{\underline{165}}$

C i.) $f(x) = 2x - 1$ $g(x) = x^2 + 5x - 2$.

$g(f(x) = g(2x - 1)$

$= (2x - 1)^2 - 5(2x - 1) - 2$ ①

$= 4x^2 - 4x + 1 + 10x - 5 - 2$ ①

$= 4x^2 + 6x + 6$ ①

ii) $g \text{ of } (2) = 4(2)^2 + 6(2) + 6$ ①

$= 16 + 12 + 6$

$= 34$ ①

3. Matrix Eqn. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 33 \\ 2 \end{pmatrix}$ ①

$X = \frac{Dx}{D}$ $y = \frac{Dy}{D}$ $Z = \frac{D_z}{D}$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & -2 \end{vmatrix} = (6+8) - (-4 - 2) + (-4 + 9) = 35$ ①

$Dx = \begin{vmatrix} 4 & 1 & 1 \\ 33 & -3 & 4 \\ 2 & -2 & -2 \end{vmatrix} = 4(6 + 8) - (-66 - 8) + (-66 + 8) = 70$ ①

$x = \frac{70}{35} = 2$ ①

$Dy = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 33 & 4 \\ 3 & 2 & -2 \end{vmatrix} = (-66 - 8) - 4(-4 - 12) + (4 - 99) = -105$ ①

$y = \frac{-105}{35} = 3$ ①

$= (-6 + 66) - (4 - 99) + 4(-4 + 9)$

$= 175$ ①, $z = \frac{175}{35} = 5$ ①

b) $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1- & -1 \\ 3 & 2 & 1 \end{pmatrix}$

Co – factors $A_{11} = +(-1 + 2) = 1$ $A_{12} = -(2 + 3) = -5$

$A_{21} = -(2 + 6) = -8$ $A_{22} = +(1 + 9) = 10$

$A_{31} = +(-2 - 3) = -5$ $A_{32} = -(-1 + 6) = -5$

$A_{13} = +(4 + 3) = 7$

$A_{23} = -(2 - 6) = 4$

$A_{33} = +(-1 - 4) = -5$

. . matrix of cofactors is $\begin{pmatrix} 1 & -5 & 7 \\ -8 & 10 & 4 \\ -5 & -5 & -5 \end{pmatrix}$

Ad joints A, Adj A $= \begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix}$

$A^{-1} = \frac{1}{141}$ Adj A where (A) = det A = 1(-1 + 2) – 2(2+3) – (4 + 3)

= 1 – 10 – 21

=-30

.

.. $A^{-1} = -\frac{1}{30}\begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix}$

For the system

$x_1 + 2x_2 - 3x_3 = 3$

$2x_1 - x_2 - x_3 = 11$

$3x_1 + 2x_2 + x_3 = -5$

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 4^{-1} \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = {}^{-1}/_{30} \begin{pmatrix} 1 & -8 & -5 \\ -5 & 10 & -5 \\ 7 & 4 & -5 \end{pmatrix} \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = {}^{-1}/_{30} \begin{bmatrix} -60 \\ 120 \\ 90 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$$

$x_1 = 2$, $x_2 = -4$, $x_3 = -2$

5\. a) i. factors of ƒ(x) are: (x+3), (x -2) and (x-4) ①

ƒ(x) = k (x+3) (x-2) (x-4) ①

ƒ(o) = k(3) (-2) (-4) = -12

$\Rightarrow$ 24k = -12

$K = -{}^{1}/_{2}$ ①

Hence $f(x) = -1/2\ (x+3)\ (x-2)\ (x-4)$ ①

ii.

$$\begin{array}{r|l} & \overline{\quad 3x2 - 4x -1 \qquad\qquad} \\ x^2+2 & 3x^4\ 4x^3 +5x^{2}\,{}^{-}7x+4 \\ & \underline{3x^4 \qquad +6x2 \qquad\qquad} \\ & \qquad -4x^3 - x^2 + 7x+4 \\ & \qquad \underline{-4x^3 \qquad -8x} \\ & \qquad\quad -x2 + x+4 \\ & \qquad\quad \underline{-x2 \quad -2} \\ & \qquad\qquad X + 6 \end{array}$$

① (after $-8x$) ① (after $X + 6$)

Hence

$3x^4 - 4x^3 + 5x^2 - 7x + 4 = (3x^2 - 4x-1)\ (x^2 + 2) + x + 6$ ①

b. $\dfrac{x^{-\frac{2}{3}}\,y^{1}/_{2}\,Z^{\frac{1}{3}}}{(x^{\frac{2}{3}}\,y^{2}\,Z^{\frac{1}{3}})\frac{1}{3}} \qquad = \dfrac{x^{-\frac{2}{3}}\,y^{\frac{1}{2}}\,Z^{\frac{1}{3}}}{x^{2}/_{9}\,y_{\frac{2}{3}}\,Z_{1/9}}$

$= X^{-2}/_{3}\,{}^{-2}/_{9}\ y^{1}/_{2}{}^{-2}/_{3}\ Z^{1}/_{3}\ -{}^{1}/_{9}$ ①

$= x^{-8}/_{9}\ y^{-1}/_{6}\ z^{2}/_{9}$ ①

$$\begin{array}{l} = Z_{2/9} \\ \quad X^{8/}{}_{9}\ Y^{-1/}{}_{6} \end{array}$$ ①

b) $\dfrac{a^2\,b^{1/}{}_{2}\,c^{1/}{}_{4}\ \ a^{1/}{}_{4}\,b^{1/4}}{\sqrt[3]{a^2\,b^3\,c^{1/}{}_{2}}} \quad = \dfrac{a^2\,b^{1}/_{2}\,c^{1}/_{4}\ a^{1/}{}_{4}\,b^{1}{}_{/4}}{a^{\frac{2}{3}}\ b\ c^{\frac{1}{6}}}$ ①

$= a^{\,2/_{1}+1/4\,-2/3}\ \ b^{\,\frac{1}{2}\,+1/4\,-1}\ \ c^{\,\frac{1}{4}\,-1/6}$

$= a^{\frac{24+3+8}{12}}\ \ b^{\frac{2+1-4}{4}}\ \ c^{\frac{6-4}{24}}$

$= a^{-19/12}\ b^{-1/4}\ c^{-1/12}$ ①

$= \dfrac{1}{a^{19}/_{12}\ b^{1}/_{4}\ C^{1}/_{12}}$ ①

c) i. $3x^2 - 7x - 8 = 0$

$$x = \frac{b + \sqrt{b^{2\,-}\,4ac}}{2a} \qquad , a = 3, b = -7, c = -8$$

$$x = \frac{-(-7) + \sqrt{(-7)^2 - (4)(3)(-8)}}{(2)(3)} \quad ①$$

$$= \frac{7 + \sqrt{49 + 96}}{6}$$

$$= \frac{7 + \sqrt{145}}{6} = 3.1736 \quad \text{or } x = \frac{7\ \sqrt{145}}{6} = 0.8403 \quad ①$$

ii. $x^2 - 7x + 12 = 0$

$x^2 - 7x = -12$

$x^2 - 7x + \left(\frac{7}{2}\right)^2 = -12 + \left(\frac{7}{2}\right)^{L}$ ①

$\left(x - \frac{7}{2}\right)^2 = -12 + 12.25$

$x - \frac{7}{2} = + \sqrt{0.25}$ ①

$x - \frac{7}{2} = \pm\, {}^{1}/_{2}$

$x = \frac{7}{2} + \frac{1}{2} = 4$ or $x = \frac{7}{2} - \frac{1}{2} = 3$