MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND INTEGRAL CALCULUS

UNIVERSITY ORDINARY EXAMINATION

2023/2024 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR DIPLOMA IN INFORMATION AND COMMUNICATION TECHNOLOGY

AMM 057: LINER ALGEBRA

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer Question one and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) What is a rank of a matrix? (1mark)

b) Find the rank of matrix A= $\begin{pmatrix} 1 & 2 & 4 & 4 \\ 3 & 4 & 8 & 0 \end{pmatrix}$ (3 marks)

c) Let there be two vectors a=(6, 2, -1) and B=(5, -8, 2)

i. Find the dot product of the vectors. (2 marks)

ii. Find the length of vector **AB.** (2 marks)

iii. Find the angle between the vectors. (4 marks)

iv. Determine whether the vectors (1,2 and (-5,3) are linearly dependent. (2 marks)

d) Given A= $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ and B=$\begin{pmatrix} 3 & 4 & 3 \\ 0 & 2 & 9 \\ 4 & 1 & 1 \end{pmatrix}$. Find BA$^{/}$A. (4 marks)

e) Use the Cramer rule to solve the following simultaneous equation. (4 marks)

$$12x + 3y = 15$$

$$2x - 3y = 13$$

f) Let A = $\begin{pmatrix} 5 & -3 \\ 2 & 2 \end{pmatrix}$ find A^{-1} (3 marks)

g) Solve the following simultaneous equation using the Gaussian elimination method.

(5 marks)

$$2x + y + 2z = 10$$

$$x + 2y + z = 8$$

$$3x + y - z = 2$$

SECTION TWO: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) What is linear transformation? State the conditions for linear transformation. (3marks)

b) Let M= $\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$

i. Write an expression for $T_{m.}$ (2 marks)

ii. Find T_m (1,0) and T_m (0,1). (4 marks)

iii. Find all points (x, y) such that T_m(x, y) = (1, 0). (2 marks)

c) Find the distance between the vectors (2, 3, 5) and (2, 0, -9). (4 marks)

d) Find the cross product of the vectors (2, 3, 5) and (2, 0, -9). (3 marks)

e) Find the sum of the vector (2, 3, 5) and (2, 0, -9). (2 marks)

QUESTION THREE (20 MARKS)

a) Given $P=\begin{pmatrix}1 & 2\\0 & 4\end{pmatrix}$ $Q=\begin{pmatrix}2 & 0\\1 & 3\end{pmatrix}$ and $R=\begin{pmatrix}3 & 0\\2 & 2\end{pmatrix}$

Find

i. 3P – 2(Q +R) (3 marks)

ii. $P^2 + {}^1\!/_2\, Q$ (3 marks)

iii. $Q^{-1}R + 2P$ (5 marks)

b) Musa spent sh. 207 to buy seven exercise books and four pens while Mary spent sh.165 to buy five exercise books and five pens of the same type.

i. Form a simultaneous equation. (2 marks)

ii. Use the matrix methods to find the cost of one exercise book and one pen. (5 marks)

iii. Find the cost of buying ten such exercise books and two pens. (2 marks)

QUESTION FOUR (20 MARKS)

a) What is the inverse of the transformation $F: R^2 \to R^2$ given by F(x, y) = (x +3y, x +5y)?

(4 marks)

b) Find a linear transformation $R^2 \to R^2$ that maps (1, 1) to (-1, 4) and (-1, 3) to (-7, 0).

(5 marks)

c) Find the angle between $\boldsymbol{a}$ = i +4j +8k and **b** = 5i +4j +3k (3 marks)

d) Two cinema theatres A and B carry 700 people each. Each of them carries 300 people upstairs and 400 downstairs. Theatre A charges sh150 for upstairs and sh.100 for downstairs. Theatre B charges sh.140 upstairs and sh. 90 downstairs. Using matrix, calculate the total collections for each theatre during a show when all the seats are booked. (4 marks)

e) If matrix $A=\begin{pmatrix}1 & 2\\-5 & 2\end{pmatrix}$ and $B=\begin{pmatrix}6 & 8\\5 & 1\end{pmatrix}$. Find matrix C if A=CB. (4 marks)