MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2023/2024 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND ECONOMICS

AMS 331: PROBABILITY AND STATISTICS IV

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer Question one and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) A fair die is tossed 12 independent times. Determine the probability of the following configuration (4 marks)

Face	1	2	3	4	5	6
No. of occurrence	2	3	0	2	4	1

b) Define the following

i. Probability generating function (2marks)

ii. Characteristic function (2marks)

iii. Let x be a random variable with generating form P(s) find the generating functions of x+1 (2marks)

c) A bank teller serves customers standing in the queue one by one. Suppose that the service time x_1 for customer I has mean $\text{E}(x_1) = 2$ minutes and $\text{var}(x_1) = 1$ assuming that the service time for different bank customers are independent. Let y be the total time the bank teller spends serving 50 customers. Find $P(90 < y < 110)$ (4marks)

d) Let $X \sim Bin(n, p)$. Using Markovs inequality find an upper bound $P(x \geq \alpha n)$ where $p(< \alpha < 1)$. Hence evaluate the bound for p= $\frac{1}{2}$ and α= $\frac{3}{4}$ (4marks)

e) Let$y = (y_1, y_2, y_3)$be a random vector which is normally distributed, $N_3(\mu, \varepsilon)$with mean vector and covenance matrix given below

$$\mu = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} and\ \varepsilon = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 10 \end{pmatrix}$$

Find the joint distribution given

$Z_1 = y_1 + y_2 + y_3$ and $Z_1 = 3y_1 + y_2 - 2y_3$ (5marks)

f) Two random variables x and y have a joint pdf

$$f(x, y) = \begin{cases} kx, & x \leq y \leq x \leq 1 \\ 0, & elsewhere \end{cases}$$

i. Evaluate the constant k (3marks)

ii. Obtain the marginal distribution of x and y and show that these random variables are not independent (4marks)

SECTION TWO: ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Let $X \sim Bin(n, p)$. Use Chebychev's inequality to find an upper bound on $P(x \geq \alpha n)$ where $p(< \alpha < 1)$. Hence evaluate the bound for p= $\frac{1}{2}$ and α= $\frac{3}{4}$ (4marks)

b) Suppose that x has a binomial distribution with parameter n and p. Obtain the pgf of x and hence find the mean and variance of x. (10marks)

c) Two tetrahedral dice are rolled together once. If x is the number facing up; prove that $p(x - 7| \geq 3) \leq 35/54$ (6marks)

QUESTION THREE (20 MARKS)

a) State and proof the weak law of a large numbers (6marks)

b) If the variable x_p assumes the value $2^{p-2logp}$ with probability p=1,2,3……

Examine if the weak law of large numbers holds (4marks)

At a particular gas station, gasoline is stocked in a bulk tank each week. Let random variable x denote the proportion of tank's capacity that is stocked in a given week and let y denote the proportion of the tanks capacity that is sold in the same week. Note that the gas station cannot sell more than what was stocked in a given week, implying that the value of y cannot exceed the value of X. Possible pdf x and y is given by

$$f(x) = \begin{cases} 3x, & x \le y \le x \le 1 \\ 0, & elsewhere \end{cases}$$

i. Obtain the joint cdf of x and y at the point (x, y) = $({}^{1}/_{2}, {}^{1}/_{3})$ (5marks)

ii. Find the probability that the amount of gas sold is less than the half the amount that is stocked in a given week (5 marks)

QUESTION FOUR (20 MARKS)

a) State and proof the central limit theorem. (10Marks)

b) Find the characteristic function of the exponential random variable x_1 where

$$f(x) = \begin{cases} e^{-ax}, & x \ge 0, a > o \\ 0, & elsewhere \end{cases}$$

Hence compute the mean and variance (10marks)