MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2023/2024 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

AMS 330: PROBABILITY AND STATISTICS 111

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question one and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Give a brief explanation of what is meant by

   i) The joint probability mass function of the random variable X and Y (2 marks)

   ii) The stochastic independence of two random variables X and Y (2 marks)

b) Let X_1 and X_2 have the joint probability density given by

$$f(x_1, x_2) = f(x) = \begin{cases} k(1-x_2), & 0 \le x_1 \le x_2 \le 1 \\ 0, & elsewhere \end{cases}$$

   i) Find the value of K (3 marks)

   ii) Find the probability of $P(x_1 \le 0.75, x_2 \ge 0.5)$ (4 marks)

c) A machine engine part produced by a company is claimed to have diameter variance no larger than 0.0002 diameter (measured in inches). A random sample of 10 parts gave a sample variance of 0.0003. Test, at the 5% level, Ho: $\sigma^2 = 0.0002$ against Ho: $\sigma^2 > 0.0002$ (5 marks)

d) (i) Define order statistics (2 marks)

   ii) Find the probability density function of $X_{(n)}$ (4 marks)

e) The joint probability density function of X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} 3x_1, & 0 \le x_2 \le x_1 \le 1 \\ 0, & elsewhere \end{cases}$$

Find:

   i) Cov (x_1, x_2) (5 marks)

   ii) Correlation coefficient (3 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) (i) Let X and Y be jointly normal random variable with parameters $\mu_x, \sigma_x^2\ \mu_y, \sigma_y^2\ and\ \rho$. Find the conditional distribution of Y given $X = x$ (7 marks)

   (ii) Suppose x and y have bivariate normal distribution with parameters $\mu_1 = \mu_2 = 2, \delta_1 = \delta_2 = 2\ and\ \rho = 3/5$

   Calculate P(Y> 4 $|X = 3)$ (6 marks)

b) If two Random variables x and y have the joint probability distribution function $P(x, y) = \begin{cases} \frac{1}{30}(x+y)\ x = 0,1,2,3\ and\ y = 0,1,2 \end{cases}$

i) Show that $P(x, y)$ satisfy the properties of a discrete joint distribution function (2 marks)

ii) Find $F(2,1)$ (3 marks)

iii) Find the Marginal distribution of X (2 marks)

QUESTION THREE (20 MARKS)

a) Consider two random samples X_1 and X_2 of sizes 10 and 20 with sample variances given by 0.0003 and 0.0001 respectively. Assuming that the populations from the samples have been drawn, are normal determine whether the variance of the first population is significantly greater than the second one. Take $\alpha=0.05$ (5 marks)

b) Derive the conditional distribution of x_1 given x_2. If x_1 and x_2 are jointly trinomial distributed. (8 marks)

c) A continuous random distribution of X has the probability density function

$f(x)=\begin{cases} \lambda x e^{-\lambda x}, & x > 0 \\ 0, & elsewhere \end{cases}$. Find moment generating function. (7 marks)

QUESTION FOUR (20 MARKS)

a) Suppose the joint distribution of x_1 and x_2 is given by

$$f(x_1, x_2)=\begin{cases} 2(1 - x_1), & 0 \le x_1 \le 1, 0 \le x_2 \le 1 \\ 0, & elsewhere \end{cases}$$

i) Find the density function of the variable $U=X_1X_2$ (5 marks)

ii) Find E(U) (4 marks)

iii) Var(U) (4 marks)

b) In an experiment to test two procedures, the following information was obtained standard procedure $n_1 = 9$, Mean $x_1 = 35.22$ seconds and $\sum_{i=1}^{9}(x_{1i} - x)^2 = 195.56$ *and* $n_2 = 9$, Mean $x_2 = 31.56$ seconds and $\sum_{i=1}^{9}(x_{2i} - x)^2 = 160.22$

Test the hypothesis that the two population have the same mean. Take $\alpha = 0.05$ level of significance. (5 marks)

c) Determine whether Random variables X and Y are independent if $f(x, y)=\begin{cases} 2e^{-x-y}, & 0 < x < y < infinity \\ 0, & elsewhere \end{cases}$ (2 marks)