MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2021/2022 ACADEMIC YEAR

FIRST YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN …………………………..

AMM 204: DIFFERENTIAL AND INTEGRAL CALCULUS

DURATION: 2 HOURS

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a. Define the following terms
   i. A function (1 mark)
   ii. Range (1 mark)

b. State the domain of
   i. F(x) = $\frac{4}{X-5X+6}$ (2 mark)
   ii. Y(x) = $2x^2 - 5x + 1$ (1 marks)

c. Evaluate the following limits
   i. Lim (t→3) $\frac{t^2+6t+9}{t^2-9}$ (3 marks)

   ii. Lim (x→ infinity) $\frac{2x^4+x^2+8x}{-5x^4+7}$ (3 marks)

d. Find the values of the constantans in the equations below if f(x) is continuous everywhere in the real number line

$$F(x) = \begin{cases} -4x \ \ if \ \ x < 1 \\ ax + b \ if \ 1 \le x < 4 \\ 32 \ if \ x \ge 4 \end{cases}$$ (5 marks)

e. Use the first principle of differentiation to find y'(x) given that y(x) = sin x (4 marks)

f. Find $\frac{dy}{dx}$ given that
   i. y(x) = tan $\sqrt{5x^2 + 3x + 2}$ (4 marks)

   ii. y(x) =ln ($3\sqrt{x^3 - 3x + 1}$ (3 marks)

g. Evaluate $\int x^2(3x - 5x^2)^5$ dx (3 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a. Given that y(x) = $\frac{U(X)}{V(X)}$ use the first principle of differentiation to show that (8 marks)

$\frac{dy}{dx} = \frac{U'(x)V(X)-U(X)V'(X)}{(V(X))^2}$ Hense find $\frac{dy}{dx}$ given that $\frac{1-x^2-4x^3}{-x^2+4x+1}$

b. Determine the values of constants A and B so that the following function is continuous everywhere on the real number line. (8 marks)

$$F(x) = \begin{cases} A(1-\cos x)/(\sin^2 x) & if\ x < 0 \\ 2x^2 - x + B & if\ 0 \leq x \leq 1 \\ (x^2+2x-3)/(x^2-1) & if\ x \geq 4 \end{cases}$$

c. A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.5 m^2/sec. At what rate is the radius decreasing if the sheet is 12m^2? (Answer to 3dp.) (4 marks)

QUESTION THREE (20 MARKS)

a. Evaluate: by partial fractions

(a) $\int \frac{5}{(2x+1)(x-2)}$ dx (5 marks)

(b) $\int \frac{5x+2}{3x+1}$ dx by first separating $\frac{5x+2}{3x+1}$ into partial fraction (3 marks)

b. A manufacturer wants to design an open box having square base and surface area of 108m^2. Find the dimensions of the box that will give maximum volume. (5 marks)

c. Estimate $82^{1/4}$ using linear approximation (4 marks)

Find $\frac{dy}{dx}$ for $xy + x^2 - 3xy^2 = y^{\frac{1}{3}}$ (3 marks)

QUESTION FOUR (20 MARKS)

a. Identify and classify the stationary points of the following functions y(x) = $x^3 - \frac{3}{2}x^2$ (7 marks)

b. In marketing a certain item, a business has discovered that the demand for a unit item is represented by p(x) = $\frac{60}{\sqrt{x}}$

The cost of producing x item is given by C(x) = 0.6x + 6000. Find the price per unit that will yield maximum profit (5 marks)

c. Approximate the value of the following integrals, use trapezoidal rule with 9 ordinates correct to 2d.p. (4 marks)

$\int_2^4 \frac{5\ln 2x}{2+\ln 2x}\,dx$

d. Find the error incurred if the following integral is estimated by Simpson's rule with n = 4

$\int_1^2 \frac{1}{x}\,dx$ (4 marks)