MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY POSTGRADUATE EXAMINATION

2020/2021 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN STATISTICS

AMS 616 – STATISTICAL INFERENCE

DURATION: 3 HOURS

Instructions to candidates:

1. Answer Any Four questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

QUESTION ONE (25 MARKS)

a) Define the following terms giving a relevant example in each case:

i. Point estimation (2 marks)

ii. Standard error (2 marks)

iii. Exponential family (2 marks)

iv. Sufficient statistics (2 marks)

v. Completeness (2 marks)

b) Let x denote the proportion of allotted time that a randomly selected student spends working on a certain aptitude test. Supposed the pdf of x is

$$f(x;\theta) = \begin{cases} (\theta+1)x^{\theta} & 0 \leq x \leq 1 \\ 0 & Otherwise \\ where -1 < \theta \end{cases}$$

A random sample of ten students yields the following data: 0.92, 0.79, 0.90, 0.65, 0.86, 0.47, 0.73, 0.97, 0.94, 0.77

i. Use the method of moments to obtain an estimator of θ then compute the estimate for this data. (5 marks)

ii. Obtain the maximum likelihood estimator of θ then compute the estimator for the given data. (5 marks)

c) Let $x_1, x_2 \ldots\ldots, x_n$ be a random sample from a distribution with pdf

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & for\ 0 < x < 1, \theta > 0 \\ 0 & elsewhere \end{cases}$$

Find a function of θ for which there exists an unbiased estimator whose variance attains the Cramer-Rao locker bound. (5 marks)

QUESTION TWO (25 MARKS)

a) State and prove the Cramer-Rao inequality. (12 marks)

b) Given the pdf

$$f(x;\theta) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty$$

Find the Cramer-Rao lower bound for θ (13 marks)

QUESTION THREE (25 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be a random sample from the density $f(x|\theta) = {}^{1}/_{\theta}\,; 0 < x < \theta$.

Estimate the loss function $l(t;\theta) = \frac{(t-\theta)^2}{\theta^2}$

Assume that Θ has the uniform distribution between 0 and 1. Find the posterior distribution of Θ. (10 marks)

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from the density

$$f(x|\theta) = \theta x^{\theta-1}; 0 < x < 1, \theta > 0$$

Assume that the prior distribution of Θ is given by $q(\theta) = \frac{1}{\Gamma(r)}\lambda^r \theta^{r-1} e^{-\theta\lambda}; 0 < \theta < \infty$

where r and λ are known

i. Derive the posterior distribution of Θ. (7 marks)

ii. Find the Bayes estimator of θ with respect to the given gamma prior distribution using a squared error loss function. (8 marks)

QUESTION FOUR (25 MARKS)

a) Using the pivotal quantity method, obtain the confidence interval for the:

i. Mean when σ^2 is unknown. (4 marks)

ii. Variance when μ is unknown (5 marks)

iii. Difference in two means (5 marks)

b) A sample was drawn from each of five populations assumed to be normal with the same variance. The values of s^2 and n in the sample size were

s^2	40	30	20	42	50
n	6	4	3	7	8

Find a 98% confidence limits for the common variance. (3 marks)

c) To test two promising new lines of hybrid corn under normal farming conditions, a seed company selected 8 farms at random in Kitale and planted both lines in experimental plots on each farm. The yields (converted to bushels per acre) for the 8 locations were

Line A	86	87	56	93	84	93	75	79
Line B	80	79	58	91	77	82	74	66

Assuming that the two yields are jointly normally distributed, estimate the difference between the mean yields by a 95% confidence interval. (8 marks)

QUESTION FIVE (25 MARKS)

a) Derive the generalized likelihood ratio test for the equality of several variances drawn from independent normal populations. (4 marks)

b) Illustrate the test of the equality of two multinomial distributions. (3 marks)

c) A cigarette manufacturer sent to each of two laboratories presumably identical samples of tobacco. Each made five determinations of the nicotine content in milligrams as follows;

   i. 24, 27, 26, 21, 24

   ii. 27, 28, 23, 31, 26

   Were the two laboratories measuring the same thing? (Assume normality and a common variance). (3 marks

d) Given the samples (1.8, 2.9, 1.4, 1.1) and (5.0, 8.6, 9.2) from normal populations, test whether the variances are equal at the 0.05 level. (3 marks)

e) Of 64 offspring of a certain cross between guinea pigs, 34 were red, 10 were black and 20 were white. According to the genetic model, these numbers should be in the ratio 9/3/4. Are the data consistent with the model at the 0.05 level? (4 marks)

f) A syrup supposed to have some effect in preventing colds was tested on 500 individuals and their records for 1 year were compared with the records of 500 untreated individuals as follows:

	No colds	One cold	More than one cold
Treated	252	145	103
Untreated	224	136	140

   Test at the 0.05 level whether the two trinomial populations may be regarded as the same. (4 marks)

g) Gilbey classified 1725 school children according to intelligence and apparent family economic level. A condensed classification follows:

	Dull	Intelligent	Very capable
Very well clothed	81	322	233
Well clothed	141	457	153
Poorly clothed	127	163	48

   Test for independence at the 0.01 level. (4 marks)